



Mathematics Education and Graph Theory

Proceedings of International Seminar
on Mathematics Education and Graph Theory
June 9, 2014

Editors:
Mustangin
Abdul Halim Fathani

MATHEMATICS EDUCATION AND GRAPH THEORY

Proceedings of International Seminar on Mathematics Education and Graph Theory

© Department of Mathematics Education Faculty of Teacher Training and Education
Islamic University of Malang, 2014

*These proceedings contain the full texts of paper and talks presented
in the International Seminar on Mathematics Education and Graph Theory
on June 9, 2014*

Reviewers

Surahmat Supangken (UNISMA, Indonesia)
Abdur Rahman As'ari (UM, Indonesia)
Kiki Ariyanti Sugeng (UI, Indonesia)
Sunismi (UNISMA, Indonesia)
Akhsanul In'am (UMM, Indonesia)

Editors

Mustangin
Abdul Halim Fathani

Layouter

Teguh Permadi

First published, 2014

ISBN 978-602-71141-0-4

Published by



Unit of Publication
Department of Mathematics Education
Faculty of Teacher Training and Education
Islamic University of Malang (UNISMA)
Malang, East Java, Indonesia
Phone +62341- 551932, 551822.
Fax +62341-552249
<http://www.unisma.ac.id>

TABLE OF CONTENTS

Preface __ iii

Table of Contents __ v

MATHEMATICS EDUCATION: THEORETICAL (CONCEPTUAL) ARTICLES

OPTIMIZING PROBLEM SOLVING ACTIVITY FOR TEACHING MATHEMATICAL THINKING

Abdur Rahman As'ari 3–9

CONSTRUCTION THEORY OF CRITICAL THINKING AS PROCESS TOWARDS REFRACTION THINKING IN MATHEMATICS

Anton Prayitno, Akbar Sutawidjaja, Subanji, Makbul Muksar 10–16

CONCEPT IMAGE AND CONCEPT DEFINITION OF A STUDENT'S CONCEPT UNDERSTANDING

Budi Nurwahyu 17–26

DEVELOPMENT OF TACIT KNOWLEDGE DIMENTION FOR MATHEMATICS NOVICE TEACHERS

Edy Bambang Irawan 27–31

DEVELOPING THE STUDENT'S MATHEMATICAL REPRESENTATION AND ABSTRACTION ABILITY THROUGH GUIDED DISCOVERY LEARNING

Eka Setyaningsih 32–38

IDENTIFICATION THE UNI-CONNECTED MATHEMATICAL THINKING PROCESS IN MATH PROBLEM SOLVING

Elly Susanti, I Nengah Parta, Tjang Daniel Chandra 39–49

BLENDED LEARNING AS A WAY TO OPTIMIZE SEMESTER CREDIT SYSTEM (SCS)

Hapizah 50–55

MAKING MATHEMATICAL CONNECTIONS IN SOLVING CONTEXTUAL MATHEMATICS PROBLEM: THEORETICAL REVIEW FROM THE PERSPECTIVE OF IQ AND GENDER

Karim 56–64

THE ROLE OF LANGUAGE, LOGIC, AND MATH IN SCIENCE AND TECHNOLOGY DEVELOPMENT

M. Kharis 65–68

REPRESENTATION OF MATHEMATICAL CONCEPT IN THE DIFFERENT PERSPECTIVE THEORY OF UNDERSTANDING

Mustangin 69–77

BUILD MATHEMATICAL KNOWLEDGE THROUGH PROBLEM-SOLVING STRATEGY

Saleh Haji 78–81

NINE STRATEGIES OF CRITICAL THINKING DEVELOPMENT: A LITERATURE REVIEW

Slamet 82–85

**MATHEMATICS EDUCATION:
RESEARCH AND DEVELOPMENT ARTICLES**

IMPLEMENTATION OF PROPS “FASKAL” TO RAISE ALGEBRA FACTORIZATION CONCEPT IN AISIYAH ORPHANAGE SUMBERSARI - JEMBER DISTRICT

Abi Suwito 89–94

THE PROFILE OF PROBLEM SOLVING ON OPEN ENDED PROBLEMS OF STUDENTS WITH INTERMEDIATE LEVEL OF MATHEMATICS COMPETENCE

Agung Deddiliawan Ismail 95–99

METACOGNITIVE AWARENESS ASPECTS IN SOLVING ALGEBRA

Akhsanul In'am 100–104

DEVELOPING SELF-RENEWAL CAPACITY SCALE BASED ON PACE MODEL

Andri Suryana 105–109

DIAGNOSTIC DIFFICULTY STUDENTS ORGANIZATION AT THE UNIVERSITY OF GUNUNG JATI IN PROVING USING MATHEMATICAL INDUCTION AND EFFORTS TO OVERCOME USING SCAFFOLDING

Azin Taufik 110–116

CHARACTERISTICS OF THINKING PROCESSES OF ELEMENTARY SCHOOL STUDENTS WITH MODERATE ABILITY IN MATHEMATICS PROBLEMS SOLVING

Baiduri 117–123

TEACHER’S KNOWLEDGE OF CONTENTS AND STUDENTS (KCS) ON QUADRILATERALS: CASE STUDY

Bettisari Napitupulu 124–134

HOW TO IMPROVE STUDENTS’ ABILITY IN QUESTIONING BY ASKING THEIR RESPONSIBILITY OF WHAT THEY LEARN

Budi Mulyono 135–139

TEACHING MATHEMATICS TO 0 – 1 YEAR OLD BABIES

Christine Wulandari 140–155

DEVELOPMENT OF MATHEMATICS LEARNING MATERIALS WITH ANCHORED INSTRUCTION MODEL FOR DISABILITY STUDENT IN INCLUSION CLASS

Dian Kristanti, Cholis Sa’dijah, Tjang Daniel Chandra 156–175

IMPLEMENTATION OF THE FIRST YEAR LESSON STUDY TO IMPROVE THE
LEARNING QUALITY IN MATHEMATICS EDUCATION STUDY PROGRAM
UNIVERSITY OF JEMBER

Dian Kurniati **176–180**

THE THINKING PROCESS OF JUNIOR HIGH SCHOOL STUDENTS
ON THE CONCEPT OF RECTANGLE REVIEWED BY THEIR COGNITIVE STYLES

Endah Budi Rahaju **181–190**

BARRIERS TO STUDENT THINKING IN SOLVING THE PROBLEM OF FACTORING
ALGEBRAIC FORM BASED ON THE LEVEL OF ALGEBRAIC THINKING
AND SCAFFOLDING

Hairus Saleh, Ukhti Raudhatul Jannah **191–207**

LEARNING MEDIA “PUTRI” IN IMPROVING TRIGONOMETRY LEARNING
OUTCOMES AT SMK (VOCATIONAL SCHOOL)

Hastini Ratna Dewi, Wiwik Sugiarti **208–217**

METHOD OF INDUCTION IN THE PROCESS OF MASTERING CALCULUS
IN MECHANICS

Hendra Gunawan **218–223**

THE REFLECTIVE THINKING STUDENT WITH LOGIC APPROACH IN PROBLEMS
SOLVING OF SPEED, DISTANCE, AND TIME

Hery Suharna, Subanji, Toto Nusantara **224–228**

DEVELOPING RESEARCH INSTRUMENTS FOR PROFILING COGNITIVE
PROCESSES OF STUDENT IN CONSTRUCTING MATHEMATICAL CONJECTURE
FROM INDUCTION PROBLEMS

I Wayan Puja Astawa **229–236**

THE PROFILE OF REASONING SCHOOLGIRLS ELEMENTARY SCHOOL WHO
HAVE HIGH MATHEMATICS ABILITY IN PROBLEMS SOLVING OF FRACTION

Iis Holisin **237–246**

A COGNITIVE LOAD OF THE SEVENTH GRADE STUDENTS IN TEACHING
MATHEMATICS AT SMP NEGERI 3 MALANG

Isbadar Nursit **247–253**

THE DEVELOPMENT OF INSTRUMENTS TO IDENTIFY CRITICAL THINKING
SKILL OF JUNIOR HIGH SCHOOL STUDENTS IN SOLVING CRITICAL THINKING
MATHEMATICS PROBLEMS

Ismail **254–265**

DEVELOPING A MATHEMATICS MODULE WITH CONTEXTUAL APPROACH AND
ISLAMIC VALUES USING ADOBE FLASH CS3 AS A MATHEMATICS LEARNING
RESOURCE FOR STUDENTS IN SMP/MTS

Mulin Nu'man, Ibrahim **266–285**

DESIGNING VIDEOS OBSERVATIONS PROJECT THROUGH SCIENTIFIC
APPROACH WITH AUTHENTIC ASSESSMENT TO INTEGRATE STUDENTS'
KNOWLEDGES IN MATHEMATICS TEACHING AND LEARNING

Nurcholif Diah Sri Lestari **286–295**

DEVELOPING SELF EFFICACY SCALE WITH THE ORIENTATION OF WEB-ASSISTED
BRAIN-BASED LEARNING

Nuriana Rachmani Dewi (Nino Adhi) **296–301**

TEACHER'S INTERACTION PROCESS IN ASSISTING STUDENTS OF SMAN 10
MALANG TO CONSTRUCT CONCEPT OF COMPREHENSION IN PROBABILITY
MATERIAL

Ratna Widyastuti **302–313**

THE ANALYSIS OF STUDENTS DIFFICULTIES IN SOLVING PROBLEMS OF SET

Rohana **314–321**

MATHEMATICS LEARNING BY MIND MAPPING METHOD

Ryan Angga Pratama, Alhamidun **322–327**

CHARACTERIZATION OF ALGEBRAIC THINKING Process OF STUDENTS
IN PATTERN GENERALIZING BASED APOS THEORY

Siti Inganah, Purwanto, Subanji, Swasono Rahardjo **328–337**

MATHEMATICAL COMMUNICATION PROFILE OF FEMALE-FIELD INDEPENDENT
STUDENT OF JUNIOR HIGH SCHOOL IN SOLVING PROBLEM

Sudi Prayitno **338–344**

THE EFFECTS OF *REALISTIC MATHEMATICS EDUCATION* AND STUDENTS'
COGNITIVE DEVELOPMENT LEVELS ON THE UNDERSTANDING OF CONCEPTS
AND THE ABILITY IN SOLVING MATHEMATIC PROBLEMS BY JUNIOR HIGH
SCHOOL STUDENTS

Sunismi **345–359**

THE IMPROVEMENT OF STUDENT UNDERSTANDS OF ADDITION
AND REDUCTION FRACTION CONCEPT THROUGH REALISTIC MATHEMATIC
EDUCATION (RME) WITH MANIPULATIVE MATERIALS IN THE 4TH GRADE
OF SDN GADANG I MALANG

Surya Sari Faradiba **360–364**

RESEARCH INSTRUMENT DEVELOPMENT OF STUDENTS' REASONING PROCESS
IN PROVING THEOREM

Susanah **365–377**

PROFILE OF STUDENT'S INTUITION IN THE ANALISYS OF THE VAN HIELE LEVEL
IN GEOMETRY PROBLEM SOLVING

Susilo Bekti **378–387**

STUDENT'S FOLDING BACK WHICH HAS A TENDENCY ON CONCEPTUAL
KNOWLEDGE IN UNDERSTANDING LIMIT DEFINITION

Susiswo **388–403**

DEVELOPING TEACHING REALISTIC MATHEMATIC INTERACTIVE HANDBOOK
ON STATISTICS SETTING ON ISLAMIC BOARDING SCHOOL OF IX GRADE MTs

Suwarno **404–413**

INSTRUCTIONAL MATERIALS DEVELOPMENT ON CALCULUS II COURSE USING
MATHEMATICS MOBILE LEARNING (MML) APPLICATION

Sunismi, Abdul Halim Fathani **414–429**

PATTERN AND STRUCTURE MATHEMATICS AWARENESS CONTRIBUTED
TO NUMBER SENSE EARLY CHILDHOOD

Timbul Yuwono **430–437**

MIDDLE SCHOOL STUDENTS' COVARIATIONAL REASONING IN CONSTRUCTING
GRAPH OF FUNCTION

Ulumul Umah, Abdur Rahman As'ari, I Made Sulandra **438–448**

THE ANALYSIS ON STUDENTS' ERRORS IN SOLVING MATHEMATICAL WORD
PROBLEMS OF CUBE AND BLOCK MATERIALS BASED ON THE STAGES OF
NEWMAN'S ERROR ANALYSIS

Umi Farihah, Moh Nashihudin **449–457**

SIGNIFICANCE TRAINING OF PEDAGOGICAL AND PROFESSIONAL COMPETENCY
THROUGH PEER LESSON METHOD IN DISCRETE MATHEMATICS SUBJECT

Wasilatul Murtafiah **458–468**

THE DEVELOPMENT OF MATHEMATICS E-PORTFOLIO ASSESSMENT MODEL FOR
SENIOR HIGH SCHOOL

Zainal Abidin, Sikky El Walida **469–476**

PROCESS OF SPATIAL REASONING ON VOCATIONAL STUDENT HIGH ABILITY
IN CONSTRUCTION CUBE (CASE STUDY ON STUDENTS WHO HAVING HIGH
SPATIAL ABILITY)

Zuraidah **477–488**

GRAPH THEORY

GRACEFUL LABELING ON BAT GRAPH $B_i(n, r, s)$

Annisa Dini Handayani, Kiki A Sugeng **491–494**

THE ANALYSIS OF AIR CIRCULATION ON COFFEE PLANTATION BASED
ON THE LEVEL OF PLANTS ROUGHNESS AND DIAMOND LADDER
GRAPHCROPPING PATTERN USING FINITE VOLUME METHOD

Arif Fatahillah, Dafik, Ervin Eka Riastutik, Susanto **495–498**

DEVELOPING MST, TSP, AND VRP APPLICATION

Darmawan Satyananda **499–508**

SUPER (a,d)-VERTEX ANTIMAGIC TOTAL LABELING ON DIRECTED CYCLE GRAPH

Devi Eka Wardani Meganingtyas, Dafik, Slamini **509–515**

SPECTRUM OF ANTIADJACENCY MATRIX OF SOME UNDIRECTED GRAPHS

Fitri Alyani, Kiki A Sugeng **516–519**

THE LOCATING CHROMATIC NUMBER OF STRONG PRODUCT OF COMPLETE GRAPH AND OTHER GRAPH

I. A. Purwasih, E. T. Baskoro, M. Bača, H. Assiyatun, D. Suprijanto **520–523**

THE CHARACTERISTICS OF CRITICAL SET IN EDGE MAGIC TOTAL LABELING ON BANANA TREE GRAPH

Irham Taufiq, Triyani, Siti Rahmah Nurshiami **524–527**

VERTEX COLORING BY TOTAL LABELINGS OF SUN, WHEEL AND PRISM GRAPHS

Isnaini Rosyida, Widodo, Ch. Rini Indrati, Kiki A. Sugeng **528–533**

ON DISCONNECTED RAMSEY $(3K_2, K_3)$ -MINIMAL GRAPHS

Kristiana Wijaya, Edy Tri Baskoro, Hilda Assiyatun, Djoko Suprijanto **534–537**

TEACHING COMBINATORIAL GAMES INTERACTIVELY USING MAPLETS

Loeky Haryanto, Arnensih Alimuddin **538–546**

THE AIR FLOW ANALYSIS OF COFFEE PLANTATION BASED ON CROPS PLANTING PATTERN OF THE TRIANGULAR GRID AND SHACKLE OF WHEEL GRAPHS BY USING A FINITE VOLUME METHOD

M. Nurrohim, Dafik, Arif Fatahillah, Moch. Avel Romanza P., Susanto **547–550**

ON THE DOMINATION NUMBER AND CHROMATIC NUMBER OF FLAKE GRAPH

Mohammad Nafie Jauhari **551–554**

SUPER (a,d) -EDGE ANTIMAGIC TOTAL LABELING OF SNAIL GRAPH

Novian Riskiana Dewi, Dafik, Susi Setiawani **555–558**

TOTAL EDGE IRREGULARITY STRENGTH OF LAMPION GRAPH

Nuris Hisan Nazula, Slamin, Dafik **559–562**

SUPER (a,d) -EDGE-ANTIMAGIC TOTAL LABELING OF UFO GRAPH

Reni Umilasari, Dafik, Slamin **563–568**

CHARACTERISTIC STUDIES OF SOLUTION THE MULTIPLE TRIP VEHICLE ROUTING PROBLEM (MTVRP) AND ITS APPLICATION IN OPTIMIZATION OF DISTRIBUTION PROBLEM

Sapti Wahyuningsih, Darmawan Satyananda **569–578**

ONE TOUCH DRAWING FOR ANDROID-BASED GRAPH THEORY LEARNING

Sikky El Walida **579–582**

VERTEX MAGIC TOTAL LABELING ON SUNS DIGRAPH

Yuni Listiana, Darmaji, Slamin **583–589**

VERTEX MAGIC TOTAL LABELING ON SUN DIGRAPHS

Yuni Listiana¹, Darmaji², Slamin³

¹Program Study of Magister Mathematics Sepuluh Nopember Institute of Technology Surabaya

²Departement of Mathematics Sepuluh Nopember Institute of Technology Surabaya

³Program of System Information University of Jember

¹yuni.listiana7@gmail.com, ²darmaji@matematika.its.ac.id, ³slamin@unej.ac.id

Abstract

Digraph $D(V, A)$ is a graph which each of edge has orientation called arc. Notice that a set of vertices and a set of arcs on D respectively is denoted by $V(D)$ and $A(D)$, with $s = |V(D)|$ and $t = |A(D)|$. A one-to-one map $\lambda: V \cup A \rightarrow (1, 2, 3, \dots, s + t)$ is a vertex magic total labeling if there is a constant k such that at any vertex y require to:

$$\sum_{x \in A^+(y)} \lambda(\overrightarrow{xy}) + \lambda(y) - \sum_{z \in A^-(y)} \lambda(\overrightarrow{yz}) = k$$

where $A^+(y)$ is a set of arcs that entering vertex y and $A^-(y)$ is a set of arcs that leaving vertex y . In this paper, we describe how to construct vertex magic total labeling on suns digraph. Sun digraph, \vec{S}_n , $n \geq 3$, is defined as cycle digraph by adding an orientation pendant at each vertex of the cycle. The orientation of arcs in the sun digraph follow clockwise direction.

Keywords: Digraph Labeling, Sun Digraph, Vertex Magic Total Labeling

INTRODUCTION

Let $G = (V, E)$ be a graph with $V(G)$ is a nonempty set of vertices and $E(G)$ is a set of edges (in short V and E), with $|V| = p$ and $|E| = q$. A labeling of a graph is any mapping that sends some set of graph elements to a set of numbers (usually to the positive integers). If the domain is the vertex-set or the edge-set, the labelings are called respectively *vertex-labelings* or *edge-labelings*. In this paper we deal with the case where the domain is $V \cup E$, and these are called *total-labelings*. There are many types of graph labelings, for example harmonious, cordial, graceful and antimagic. In this paper, we focus on one type of labeling called vertex-magic total labeling. Gallian (2013) outlined many more of graph labeling in a general survey of graph.

The concept of vertex magic total labeling were first introduce by MacDougall *et al.* (2002), this is an assignment of the integers from 1 to $p + q$ to the vertices and edges of G so that at each vertex the vertex label and the labels on the edges incident at that vertex add to a fixed constant. More formally, a one-to-one map λ from $V \cup E$ onto the integer $\{1, 2, \dots, p + q\}$ is a vertex-

magic total labeling if there is a constant k so that for every vertex x :

$$\lambda(x) + \sum \lambda(xy) = k$$

where the sum is over all vertices y adjacent to x . Let us call the sum of labels at vertex x as the weight of the vertex; so we require $wt(x) = k$ for all $x \in V$. The constant k is called the *magic constant* for λ . M.T.Rahim and Slamin (2012) was proved that disjoint union of sun graphs $S_{t_1} \cup S_{t_2} \cup \dots \cup S_{t_n}$, $t_i \geq 3$ for every $i = 1, 2, \dots, n$ and $n \geq 1$, has a vertex magic total labeling. In this paper, we deal to construct a vertex magic total labeling on digraph, specially is sun digraphs. Let $D = (V, A)$ is a digraph with vertex set V and arc set A with $|V| = s$ and $|A| = t$. Then, a vertex magic total labeling of digraph D is a one-to-one map that carries a set of $V \cup A$ into a set of number $\{1, 2, \dots, s + t\}$, so that the weight of the vertex require to $wt(x) = k$ for all $x \in V$.

LITERATURE REVIEW

Digraph

A digraph D consist of a non-empty finite set $V(D)$ of elements called vertices and a set $A(D)$ of ordered pair of vertices called arcs. We call $V(D)$ as the vertex set

and $A(D)$ as the arc set of D . We will often write $D(V, A)$ which means that V and A are the vertex set and arc set of D , respectively. The order (size) of D is the number of vertices (arcs) in D (Slamin, 2009). Let the order n size of D respectively are denoted as s and t . For an arc $e = (x, y)$, the first vertex x is its initial end vertex and the second vertex y is its terminal end vertex, and arc e is said to be incident out of vertex x and e is said to be incident into y . An *in-neighbour* (respectively, *out-neighbour*) of a vertex y in D is a vertex x (respectively, z) such that $(x, y) \in A$ (respectively, $(y, z) \in A$). The set of all in-neighbours (respectively, out-neighbours) of a vertex y is called the *in-neighbourhood* (respectively, the *out-neighbourhood*) of y and denoted by $N^-(y)$ (respectively, $N^+(y)$) (Dafik, 2008). In this paper, the orientation of arcs follow clockwise direction.

Vertex Magic Total Labeling

MacDougall *et al.* was introduce the concept of vertex magic total labeling (in short VMTL) on graph for first time in 2002. Hefetz *et al.* (2010) shown that an antimagic labeling of a directed graph D with s vertices and t arcs is a bijection from the set of arcs of D to the integers $\{1, \dots, t\}$ such that all s oriented vertex sums are pairwise distinct, where an oriented vertex sum is the sum of labels of all arcs entering that vertex minus the sum of labels of all arcs leaving it. Similarly, we can call that a magic labeling on digraph D is a one-to-one mapping from the set of arcs on D to the integers $\{1, \dots, t\}$ such that all s oriented vertex sums are equal.

Vertex magic total labeling on digraph D is a one-to-one map λ from $V \cup A$ onto the integers $\{1, 2, \dots, s + t\}$ if there is a constant k so that for every vertex $y \in V$ require to:

$$\sum_{x \in A^+(y)} \lambda(\overrightarrow{xy}) + \lambda(y) - \sum_{z \in A^-(y)} \lambda(\overrightarrow{yz}) = k$$

where $A^+(y)$ is an set of arcs that entering vertex y or in-neighbourhood of y and $A^-(y)$ is a set of arcs that leaving vertex y or out-neighbourhood of y . Let us call the sum of labels at vertex y as the weight of the vertex, so that we require $wt(y) = k$ for all

$y \in V$ and the constant k is called the *magic constant* for λ .

METHOD

In this paper we use several method for construct a vertex magic total labeling on sun digraphs, such as study literature for get insight of concept of vertex magic total labeling, observation for sun digraph to get the characteristic of it, construct a vertex magic labeling on sun digraph so that we get a labeling that expandable for all sun digraph with different n and find a bijective function of vertex magic total labeling on sun digraphs using pattern recognition method.

RESULT

Vertex Magic Total Labeling on Sun Digraph

Sun Digraph, denoted by $\overrightarrow{S_n}$ is a cycle digraph, $\overrightarrow{C_n}$, with adding an orientation pendant to each vertex of the cycle digraph. The sun digraph $\overrightarrow{S_n}$ consist of the vertex set $V = \{x_i | 1 \leq i \leq n\} \cup \{y_i | 1 \leq i \leq n\}$ and arc set $A = \{\overrightarrow{x_i x_{i+1}} | 1 \leq i \leq n\} \cup \{\overrightarrow{x_i y_i} | 1 \leq i \leq n\}$, with $n \geq 3$ and $i + 1$ is taken modulo n . Notice that x_i are inner vertices and y_i are outer vertices of $\overrightarrow{S_n}$. Thus, $\overrightarrow{S_n}$ has $2n$ vertices and $2n$ arcs. The following theorem describes a vertex magic total labeling on sun digraph $\overrightarrow{S_n}$, for $n \geq 3$.

Theorem 1. For $n \geq 3$, sun digraph $\overrightarrow{S_n}$ has a vertex magic total labeling with magic constant $2n + 1$.

Proof. Let s and t are order and size of $\overrightarrow{S_n}$, thus $s = 2n$ and $t = 2n$. Define total labeling $\lambda_1: V \cup A \rightarrow (1, 2, 3, \dots, s + t)$ in the following way:

$$\lambda_1(x_i) = \begin{cases} 2n + i, & \text{if } 1 \leq i \leq n - 1 \\ 4n, & \text{if } i = n \end{cases}$$

$$\lambda_1(y_i) = 2n + 1 - i, \quad \text{if } 1 \leq i \leq n$$

$$\begin{aligned} \lambda_1(\overrightarrow{x_i x_{i+1}}) &= \begin{cases} 4n - i - 1, & \text{if } 1 \leq i \leq n - 1 \\ 4n - 1, & \text{if } i = n \end{cases} \end{aligned}$$

$$\lambda_1(\overrightarrow{x_i y_i}) = i, \text{ if } 1 \leq i \leq n$$

Then we get the label vertices and arcs of sun digraph $\overrightarrow{S_n}$, that is both of $\lambda_1(A) = \{1, 2, \dots, n, 3n, 3n+1, \dots, 4n-1\}$ and $\lambda_1(V) = \{n+1, n+2, \dots, 3n-1, 4n\}$ was a set which complementary. So it is easy to verify that the labeling λ_1 is a bijection from the set $V \cup A$ onto the set $\{1, 2, 3, \dots, 4n\}$.

Let us denote the weight of the vertices x_i and y_i of $\overrightarrow{S_n}$ under the labeling λ_1 by:

$$wt_{\lambda_1}(x_i) = \lambda_1(\overrightarrow{x_{i-1} x_i}) + \lambda_1(x_i) - \lambda_1(\overrightarrow{x_i x_{i+1}}) - \lambda_1(\overrightarrow{x_i y_i})$$

And

$$wt_{\lambda_1}(y_i) = \lambda_1(\overrightarrow{x_i y_i}) + \lambda_1(y_i)$$

Then for all $i = 1, 2, \dots, n$ and $i+1$ is taken modulo n , the weight of the vertices x_i can be determined as following way:

- for $1 \leq i \leq n-1$, we have

$$wt_{\lambda_1}(x_i) = \lambda_1(\overrightarrow{x_{i-1} x_i}) + \lambda_1(x_i) - \lambda_1(\overrightarrow{x_i x_{i+1}}) - \lambda_1(\overrightarrow{x_i y_i})$$

$$= (4n - (i-1) - 1) + (2n + i) - (4n - i - 1) - (i)$$

$$= 2n + 1$$

- for $i = n$, we have

$$wt_{\lambda_1}(x_n) = \lambda_1(\overrightarrow{x_{n-1} x_n}) + \lambda_1(x_n) - \lambda_1(\overrightarrow{x_n x_1}) - \lambda_1(\overrightarrow{x_n y_n})$$

$$= (4n - (n-1) - 1) + 4n - (4n - 1) - (n)$$

$$= 2n + 1$$

By the similar way, for all $i = 1, 2, \dots, n$ and $i+1$ is taken modulo n , the weight of the vertices y_i can be determined as following way:

$$wt_{\lambda_1}(y_i) = \lambda_1(\overrightarrow{x_i y_i}) + \lambda_1(y_i)$$

$$= i + (2n + 1 - i)$$

$$= 2n + 1$$

Since the weight of vertices x_i and y_i is $wt_{\lambda_1}(x_i) = wt_{\lambda_1}(y_i) = 2n + 1$, for all $1 \leq i \leq n$, then λ_1 is a vertex magic total labeling with magic constant $2n + 1$. Thus, sun digraph $\overrightarrow{S_n}$, $n \geq 3$, has a vertex magic total labeling with magic constant $2n + 1$ is proved. \square

The example of vertex magic total labeling on sun digraph $\overrightarrow{S_n}$ is given by Figure 1.

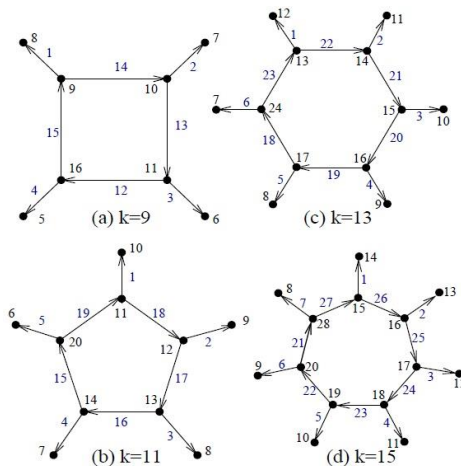


Figure 1. Vertex Magic Total Labeling on (a) $\overrightarrow{S_4}$, (b) $\overrightarrow{S_5}$, (c) $\overrightarrow{S_6}$, and (d) $\overrightarrow{S_7}$

Vertex Magic Total Labeling on Disjoint union of Sun Digraphs

The disjoint union of m copies sun digraphs $\overrightarrow{S_n}$, denoted by $m\overrightarrow{S_n}$, is defined as

digraph with vertex set $V = \{x_i^c | 1 \leq i \leq n, 1 \leq c \leq m\} \cup \{y_i^j | 1 \leq i \leq n, 1 \leq j \leq m\}$ and arc set $A = \{\overrightarrow{x_i^c x_{i+1}^c} | 1 \leq i \leq n, 1 \leq c \leq m\}$

$j \leq m\} \cup \{\overrightarrow{x_i^j y_i^j} | 1 \leq i \leq n, 1 \leq j \leq m\}$,
with $n \geq 3$ and $i + 1$ is taken modulo n .
The order and size of $m\overrightarrow{S_n}$ respectively are
 $2nm$ vertices and $2nm$ arcs. The following
theorem describes a vertex magic total
labeling on digraph $m\overrightarrow{S_n}$, for $m \geq 2$ and
 $n \geq 3$.

Theorem 2. For $m \geq 2$ and $n \geq 3$, the m -
copies of sun digraphs $m\overrightarrow{S_n}$ has a vertex
magic total labeling with magic constant
 $2mn + 1$.

Proof. Let s and t are order and size of
 $m\overrightarrow{S_n}$, thus $s = 2mn$ and $t = 2mn$. Define
total labeling $\lambda_2: V \cup A \rightarrow (1, 2, 3, \dots, s + t)$
in the following way:

- for $1 \leq i \leq n - 1$, we have

$$\lambda_2(x_i^j) = (2n + i - 1)m + j, \quad 1 \leq j \leq m$$
- for $i = n$, we have

$$\lambda_2(x_i^j) = 3mn - m + n + j, \quad i = n$$
- for $1 \leq i \leq n$ and $1 \leq j \leq m$, we have

$$\lambda_2(y_i^j) = (2n + 1 - i)m - (j - 1)$$

$$\lambda_2(\overrightarrow{x_i^j y_i^j}) = (i - 1)m + j$$

Then we get the label vertices and arcs of
 $m\overrightarrow{S_n}$, that is:

$$\begin{aligned} \lambda_2(A) = \{ & 1, 2, \dots, mn, (3n - 1)m \\ & + 1, (3n - 1)m \\ & + 2, \dots, (3n - 1) + n, (3m + 1)n \\ & + 1, \\ & (3m + 1)n + 2, \dots, 4mn \} \end{aligned}$$

and

$$\lambda_2(V) = \{mn + 1, mn + 2, \dots, (3n - 1)m, (3n$$

$$\begin{aligned} & - 1)m + n + 1, (3n - 1)m + n \\ & + 2, \\ & \dots, (3m + 1)n \} \end{aligned}$$

where $\lambda_2(A)$ and $\lambda_2(V)$ were a set which
complementary. So it is easy to verify that
the labeling λ_2 is a bijection from the set
 $V \cup A$ onto the set $\{1, 2, \dots, 4mn\}$.

Let us denote the weight of the
vertices x_i^j and y_i^j of $m\overrightarrow{S_n}$ under the labeling
 λ_2 by:

$$\begin{aligned} wt_{\lambda_2}(x_i^j) = & \lambda_2(\overrightarrow{x_{i-1}^j x_i^j}) + \lambda_2(x_i^j) - \lambda_2(\overrightarrow{x_i^j x_{i+1}^j}) \\ & - \lambda_2(\overrightarrow{x_i^j y_i^j}) \text{ and} \\ wt_{\lambda_2}(y_i^j) = & \lambda_2(\overrightarrow{x_i^j y_i^j}) + \lambda_2(y_i^j) \end{aligned}$$

Then, for all $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$
and $i + 1$ is taken modulo n , the weight of
the vertices x_i^j can be determined as
following way:

- for $j = 1$ and $1 \leq i \leq n - 1$, we have

$$\begin{aligned} wt_{\lambda_2}(x_i^1) = & \lambda_2(\overrightarrow{x_{i-1}^1 x_i^1}) + \lambda_2(x_i^1) \\ & - \lambda_2(\overrightarrow{x_i^1 x_{i+1}^1}) - \lambda_2(x_i^1 y_i^1) \\ = & (3mn + n - (m + 1) \\ & - ((i - 1) \\ & - 1)) \\ & + ((2n + i \\ & - 1)m \\ & + 1) \\ & - (3mn + n - (m + 1) \\ & - (i \\ & - 1)) - ((i - 1)m + 1) \\ = & 2mn + 1 \end{aligned}$$

- for $j = 1$ and $i = n$, we have

$$\begin{aligned} wt_{\lambda_2}(x_n^1) = & \lambda_2(\overrightarrow{x_{n-1}^1 x_n^1}) + \lambda_2(x_n^1) \\ & - \lambda_2(\overrightarrow{x_n^1 x_1^1}) - \lambda_2(x_n^1 y_n^1) \\ = & (3mn + n - (m + 1) \\ & - ((n \\ & - 1) - 1)) + (3mn - m \\ & + n \\ & + 1) - (3mn - m + n) \\ & - ((n \\ & - 1)m + 1) \\ = & 2mn + 1 \end{aligned}$$

- for $2 \leq j \leq m$ and $1 \leq i \leq n-1$, we have

$$\begin{aligned} wt_{\lambda_2}(x_i^j) &= \lambda_2(\overrightarrow{x_{i-1}^j x_i^j}) + \lambda_2(x_i^j) \\ &\quad - \lambda_2(\overrightarrow{x_i^j x_{i+1}^j}) - \lambda_2(\overrightarrow{x_i^j y_i^j}) \\ &= ((3m+j)n - (i-1)) \\ &\quad + ((2n \\ &\quad + i-1)m + j \\ &\quad - ((3m \\ &\quad + j)n - i)) \\ &\quad - ((i-1)m + j) \\ &= 2mn + 1 \end{aligned}$$

- for $2 \leq j \leq m$ and $i = n$, we have

$$\begin{aligned} wt_{\lambda_2}(x_n^j) &= \lambda_2(\overrightarrow{x_{n-1}^j x_n^j}) + \lambda_2(x_n^j) \\ &\quad - \lambda_2(\overrightarrow{x_n^j x_1^j}) - \lambda_2(x_n^j y_n^j) \end{aligned}$$

$$\begin{aligned} &= ((3m+j)n - (n-1)) \\ &\quad - (3mn \\ &\quad - m + n + j) + ((3m \\ &\quad + j)n \\ &\quad - ((n-1)m + j)) \\ &= 2mn + 1 \end{aligned}$$

By the similar way, for all $j = 1, 2, \dots, m$, $i = 1, 2, \dots, n$ and $i+1$ is taken modulo n , the weight of the vertices y_i^j can be determined as following way:

$$\begin{aligned} wt_{\lambda_2}(y_i^j) &= \lambda_2(\overrightarrow{x_i^j y_i^j}) + \lambda_2(y_i^j) \\ &= ((i-1)m + j) - ((2n \\ &\quad - i \\ &\quad + 1)m - (j-1)) \\ &= 2mn + 1 \end{aligned}$$

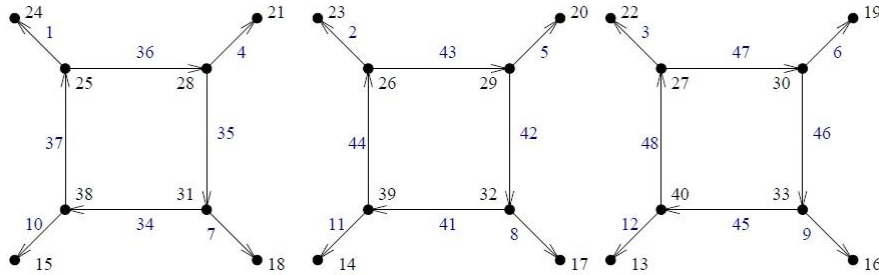


Figure 2. Vertex Magic Total Labeling on $3\overrightarrow{S_4}$ with $k = 25$

Since the weight of vertices x_i^j and y_i^j is $wt_{\lambda_2}(x_i^j) = wt_{\lambda_2}(y_i^j) = 2mn + 1$ for all $1 \leq j \leq m$ and $1 \leq i \leq n$, then λ_2 is a vertex magic total labeling with magic constant $2mn + 1$. Thus, digraph $m\overrightarrow{S_n}$, $m \geq 2$ and $n \geq 3$, has a vertex magic total labeling with magic constant $2mn + 1$ is proved. \square

The example of vertex magic total labeling on disjoint union of 3 copies of $\overrightarrow{S_4}$ is given in Figure 2. We note that if $m = 1$, then $m\overrightarrow{S_n}$ is isomorphic to $\overrightarrow{S_n}$. In this case Theorem 2 shows the vertex magic total labeling on $\overrightarrow{S_n}$ with magic constant $2mn + 1 = 2(1)n + 1 = 2n + 1$. In general we can combine Theorem 1 and Theorem 2 as follow:

Theorem 2'. For $m \in \mathbb{N}$ and $n \geq 3$, the m -copies of sun digraphs $m\overrightarrow{S_n}$ has a vertex

magic total labeling with magic constant $2mn + 1$.

Super Vertex Magic Total Labeling on Sun Digraphs

In this section we consider a super vertex magic total labeling on sun digraph. Vertex magic total labeling on G is called *super* if $\lambda(V) = \{1, 2, \dots, p\}$ (MacDougall *et al.*, 2004). Thus, vertex magic total labeling on digraph D is called *super* if $\lambda: V \rightarrow \{1, 2, \dots, s\}$ and $\lambda: A \rightarrow \{s+1, \dots, s+t\}$, for $s = |V|$ and $t = |A|$. The following theorem present super vertex magic total labeling of m -copies of sun digraph.

Theorem 3. The m -copies of sun digraphs $m\overrightarrow{S_n}$ is not super vertex magic total labeling for all $m \in \mathbb{N}$ and $n \geq 3$.

Proof. Let s and t are order and size of $m\overrightarrow{S_n}$ respectively, thus $s = 2mn$ and $t = 2mn$.

Define λ_3 is one-to-one map $\lambda_3: V \rightarrow \{1, 2, \dots, s\}$ and $\lambda_3: A \rightarrow \{s+1, s+2, \dots, s+t\}$. If $Wt_{\vec{e}_3}$ is sum of the weight of vertices, $wt_{\vec{e}_3}$, from all vertices x_i^j and y_i^j in $m\vec{S}_n$, then $Wt_{\vec{e}_3}$ is given by following way:

$$\begin{aligned} Wt_{\vec{e}_3} &= \sum_{f=s+1}^{s+t} f + \sum_{g=1}^s g - \sum_{f=s+1}^{s+t} f \\ &= \sum_{g=1}^{2mn} g \\ &= \frac{2mn(2mn+1)}{2} \end{aligned}$$

If k is magic constant for super vertex magic total labeling on $m\vec{S}_n$, then we get:

$$\begin{aligned} k &= \overline{Wt_{\vec{e}_3}} \\ &= \frac{2mn(2mn+1)}{2} \\ &= \frac{2mn}{2mn+1} \\ &= \frac{2mn+1}{2} \end{aligned}$$

for all $m \in \mathbb{N}$ and $n \geq 3$, we get magic constant k is not integer. This condition is contradiction that k is require equal to the weight of vertices, so that magic constant k must be integer. Thus, there is no super vertex magic total labeling on digraphs $m\vec{S}_n$ for all $m \in \mathbb{N}$ and $n \geq 3$. \square

Since the order and size of $m\vec{S}_n$ is $s = t = 2mn$, then we called vertex magic total labeling is *anti-super* if $\vec{e}: V \rightarrow \{s+1, s+2, \dots, s+t\}$.

Theorem 3'. *There is no anti-super vertex magic total labeling on the m -copies of sun digraphs $m\vec{S}_n$ for all $m \in \mathbb{N}$ and $n \geq 3$.*

Proof. Define \vec{e}_4 is anti-super vertex magic total labeling on sun digraphs $m\vec{S}_n$, then $\vec{e}_4: V \rightarrow \{s+1, s+2, \dots, s+t\}$ and $\vec{e}_4: A \rightarrow \{1, 2, \dots, s\}$. Let $Wt_{\vec{e}_4}$ is sum of the vertices weight, $wt_{\vec{e}_4}$, from all vertices x_i^j and y_i^j in $m\vec{S}_n$, then $Wt_{\vec{e}_4}$ is given by following way:

$$Wt_{\vec{e}_4} = \sum_{f=1}^s f + \sum_{g=s+1}^{s+t} g - \sum_{f=1}^s f$$

$$\begin{aligned} &= \sum_{g=2mn+1}^{4mn} g \\ &= \frac{2mn(4mn+2mn+1)}{2} \\ &= \frac{2mn(6mn+1)}{2} \end{aligned}$$

If k is magic constant for anti-super vertex magic total labeling on $m\vec{S}_n$, then we get:

$$\begin{aligned} k &= \overline{Wt_{\vec{e}_4}} \\ &= \frac{2mn(6mn+1)}{2} \\ &= \frac{2mn}{6mn+1} \\ &= \frac{6mn+1}{2} \end{aligned}$$

for all $m \in \mathbb{N}$ and $n \geq 3$, we get k is not integer. This condition is contradiction that k is require equal to the weight of vertices, so magic constant k must be integer. Therefore, there is no anti-super vertex magic total labeling on digraphs $m\vec{S}_n$ for all $m \in \mathbb{N}$ and $n \geq 3$. \square

CONCLUSION

We conclude this paper with the following open problem related to vertex magic total labeling on sun digraph.

Open problem 1. *Find vertex magic total labeling on disjoint union of non-isomorphic sun digraph.*

REFERENCES

- D. Hefetz, T. Mütze, J. Schwartz. 2010. On antimagic directed graphs, *Journal of Graph Theory*, **64**, Issue 3, 219–232.
- Dafik, 2008. *Handout: Pemodelan Matematika*, Jember: FKIP Universitas Jember.
- J. A. Gallian. 2013. A Dynamic Survey of Graph Labelling, *The Electronic Journal of Combinatorics*, **16**, # DS6.
- J.A. MacDougall, M. Miller and K.A. Sugeng. 2004. Super Vertex-magic total labelings of Graphs, *Proceedings of the 15th Australasian Workshop on Combinatorial Algorithms*, 222–229.
- J.A. MacDougall, M. Miller, Slamun and W.D. Wallis, 2002. Vertex-magic

- total labeling, *Utilitas Math*, **61**, 3-21.
- M.T. Rahim and Slamin , 2012. On vertex-magic total labeling of union of sun graphs, *Ars Combinatoria*, **103**, 305-310.
- N. Hartsfield and G. Ringel, 1994. *Pearls in Graph Theory*, Academic Press, San Diego.
- Slamin, 2009. *Desain Jaringan: Pendekatan Teori Graf*, Jember: Jember University Press.



Unit of Publication
Department of Mathematics Education
Faculty of Teacher Training and Education
Islamic University of Malang (UNISMA)
Malang, East Java, Indonesia

