Mathematics Education and Graph Theory

Proceedings of International Seminar
on Mathematics Education and Graph Theory
June 9, 2014

Editors:
Mustangin
Abdul Halim Fathani

MATHEMATICS EDUCATION AND GRAPH THEORY
Proceedings of International Seminar on Mathematics Education and Graph Theory

These proceedings contain the full texts of paper and talks presented in the International Seminar on Mathematics Education and Graph Theory on June 9, 2014

Reviewers
Surahmat Supangken (UNISMA, Indonesia)
Abdur Rahman As'ari (UM, Indonesia)
Kiki Ariyanti Sugeng (UI, Indonesia)
Sunismi (UNISMA, Indonesia)
Akhsanul In'am (UMM, Indonesia)

Editors
Mustangin
Abdul Halim Fathani

Layouter
Teguh Permadi

First published, 2014
ISBN 978-602-71141-0-4

Published by

Unit of Publication
Department of Mathematics Education
Faculty of Teacher Training and Education
Islamic University of Malang (UNISMA)
Malang, East Java, Indonesia
Phone +62341- 551932, 551822.
Fax +62341-552249
http://www.unisma.ac.id

TABLE OF CONTENTS

MATHEMATICS EDUCATION:
THEORETICAL (CONCEPTUAL) ARTICLES

MATHEMATICS EDUCATION:
RESEARCH AND DEVELOPMENT ARTICLES

 ISBN 978-602-71141-0-4

ISBN 978-602-71141-0-4

GRAPH THEORY

 ISBN 978-602-71141-0-4

VERTEX MAGIC TOTAL LABELING ON SUN DIGRAPHS

Yuni Listiana[1], Darmaji[2], Slamin[3]
[1]Program Study of Magister Mathematics Sepuluh Nopember Institute of Technology Surabaya
[2]Departement of Mathematics Sepuluh Nopember Institute of Technology Surabaya
[3]Program of System Information University of Jember
[1]yuni.listiana7@gmail.com, [2]darmaji@matematika.its.ac.id, [3]slamin@unej.ac.id

Abstract

Digraph $D(V, A)$ is a graph which each of edge has orientation called arc. Notice that a set of vertices and a set of arcs on D respectively is denoted by V(D) and A(D), with $s = |V(D)|$ and $t = |A(D)|$. A one-to-one map $\lambda: V \cup A \longrightarrow (1, 2, 3, \ldots, s + t)$ is a vertex magic total labeling if there is a constant k such that at any vertex y require to:

$$\sum_{x \in A^{+}(y)} \lambda(\overrightarrow{xy}) + \lambda(y) - \sum_{z \in A^{-}(y)} \lambda(\overrightarrow{yz}) = k$$

where $A^{+}(y)$ is a set of arcs that entering vertex y and $A^{-}(y)$ is a set of arcs that leaving vertex y. In this paper, we describe how to construct vertex magic total labeling on suns digraph. Sun digraph, $\overrightarrow{S_n}$, $n \geq 3$, is defined as cycle digraph by adding an orientation pendant at each vertex of the cycle. The orientation of arcs in the sun digraph follow clockwise direction.

Keywords: *Digraph Labeling, Sun Digraph,Vertex Magic Total Labeling*

INTRODUCTION

Let $G = (V, E)$ be a graph with $V(G)$ is a nonempty set of vertices and $E(G)$ is a set of edges (in short V and E), with $|V| = p$ and $|E| = q$. A *labeling* of a graph is any mapping that sends some set of graph elements to a set of numbers (usually to the positive integers). If the domain is the vertex-set or the edge-set, the labelings are called respectively *vertex-labelings* or *edge-labelings*. In this paper we deal with the case where the domain is $V \cup E$, and these are called *total-labelings*. There are many types of graph labelings, for example harmonious, cordial, graceful and antimagic. In this paper, we focus on one type of labeling called vertex-magic total labeling. Gallian (2013) outlined many more of graph labeling in a general survey of graph.

The concept of vertex magic total labeling were first introduce by MacDougall *et al*. (2002), this is an assignment of the integers from 1 to $p + q$ to the vertices and edges of G so that at each vertex the vertex label and the labels on the edges incident at that vertex add to a fixed constant. More formally, a one-to-one map λ from $V \cup E$ onto the integer $\{1, 2, \ldots, p + q\}$ is a vertex-magic total labeling if there is a constant k so that for every vertex x:

$$\lambda(x) + \sum \lambda(xy) = k$$

where the sum is over all vertices y adjacent to x. Let us call the sum of labels at vertex x as the weight of the vertex; so we require $wt(x) = k$ for all $x \in V$. The constant k is called the *magic constant* for λ. M.T.Rahim and Slamin (2012) was proved that disjoint union of sun graphs $S_{t_1} \cup S_{t_2} \cup \ldots \cup S_{t_n}$, $t_i \geq 3$ for every $i = 1, 2, \ldots, n$ and $n \geq 1$, has a vertex magic total labeling. In this paper, we deal to construct a vertex magic total labeling on digraph, specially is sun digraphs. Let $D = (V, A)$ is a digraph with vertex set V and arc set A with $|V| = s$ and $|A| = t$. Then, a vertex magic total labeling of digraph D is a one-to-one map that carries a set of $V \cup A$ into a set of number $\{1, 2, \ldots, s + t\}$, so that the weight of the vertex require to $wt(x) = k$ for all $x \in V$.

LITERATURE REVIEW

Digraph

A digraph D consist of a non-empty finite set $V(D)$ of elements called vertices and a set $A(D)$ of ordered pair of vertices called arcs. We call $V(D)$ as the vertex set

ISBN 978-602-71141-0-4

and $A(D)$ as the arc set of D. We will often write $D(V, A)$ which means that V and A are the vertex set and arc set of D, respectively. The order (size) of D is the number of vertices (arcs) in D (Slamin, 2009). Let the order n size of D respectively are denoted as s and t. For an arc $e = (x, y)$, the first vertex x is its initial end vertex and the second vertex y is its terminal end vertex, and arc e is said to be incident out of vertex x and e is said to be incident into y. An *in-neighbour* (respectively, *out-neighbour*) of a vertex y in D is a vertex x (respectively, z) such that $(x, y) \in A$ (respectively, $(y, z) \in A$). The set of all in-neighbours (respectively, out-neighbours) of a vertex y is called the *in-neighbourhood* (rspectively, the *out-neighbourhood*) of y and denoted by $N^-(y)$ (respectively, $N^+(y)$) (Dafik, 2008). In this paper, the orientation of arcs follow clockwise direction.

Vertex Magic Total Labeling

MacDougall *et al.* was introduce the concept of vertex magic total labeling (in short VMTL) on graph for first time in 2002. Hefetz *et al.* (2010) shown that an antimagic labeling of a directed graph D with s vertices and t arcs is a bijection from the set of arcs of D to the integers $\{1, \ldots, t\}$ such that all s oriented vertex sums are pairwise distinct, where an oriented vertex sum is the sum of labels of all arcs entering that vertex minus the sum of labels of all arcs leaving it. Similarly, we can call that a magic labeling on digraph D is a one-to-one maping from the set of arcs on D to the integers $\{1, \ldots, t\}$ such that all s oriented vertex sums are equal.

Vertex magic total labeling on digraph D is a one-to-one map λ from $V \cup A$ onto the integers $\{1, 2, \ldots, s + t\}$ if there is a constant k so that for every vertex $y \in V$ require to:

$$\sum_{x \in A^+(y)} \lambda(\overrightarrow{xy}) + \lambda(y) - \sum_{z \in A^-(y)} \lambda(\overrightarrow{yz}) = k$$

where $A^+(y)$ is an set of arcs that entering vertex y or in-neighbourhood of y and $A^-(y)$ is a set of arcs that leaving vertex y or out-neighbourhood of y. Let us call the sum of labels at vertex y as the weight of the vertex, so that we require $wt(y) = k$ for all $y \in V$ and the constant k is called the *magic constan*t for λ.

METHOD

In this paper we use several method for construct a vertex magic total labeling on sun digraphs, such as study literature for get insight of concept of vertex magic total labeling, observation for sun digraph to get the characteristic of it, construct a vertex magic labeling on sun digraph so that we get a labeling that expandable for all sun digraph with different n and find a bijective fungtion of vertex magic total labeling on sun digraphs using pattern recognition method.

RESULT

Vertex Magic Total Labeling on Sun Digraph

Sun Digraph, denoted by $\overrightarrow{S_n}$ is a cycle digraph, $\overrightarrow{C_n}$, with adding an orientation pendant to each vertex of the cycle digraph. The sun digraph $\overrightarrow{S_n}$ consist of the vertex set $V = \{x_i | 1 \leq i \leq n\} \cup \{y_i | 1 \leq i \leq n\}$ and arc set $A = \{\overrightarrow{x_i x_{i+1}} | 1 \leq i \leq n\} \cup \{\overrightarrow{x_i y_i} | 1 \leq i \leq n\}$, with $n \geq 3$ and $i + 1$ is taken modulo n. Notice that x_i are inner vertices and y_i are outer vertices of $\overrightarrow{S_n}$. Thus, $\overrightarrow{S_n}$has $2n$ vertices and $2n$ arcs. The following theorem describes a vertex magic total labeling on sun digraph $\overrightarrow{S_n}$, for $n \geq 3$.

Theorem 1. *For* $n \geq 3$, *sun digraph* $\overrightarrow{S_n}$ *has a vertex magic total labeling with magic constant* $2n + 1$.

Proof. Let s and t are order and size of $\overrightarrow{S_n}$, thus $s = 2n$ and $t = 2n$. Define total labeling $\lambda_1 : V \cup A \longrightarrow (1, 2, 3, \ldots, s + t)$ in the following way:

$$\lambda_1(x_i) = \begin{cases} 2n + i, & \text{if } 1 \leq i \leq n - 1 \\ 4n, & \text{if } i = n \end{cases}$$

$$\lambda_1(y_i) = 2n + 1 - i, \quad \text{if } 1 \leq i \leq n$$

$$\lambda_1(\overrightarrow{x_i x_{i+1}}) = \begin{cases} 4n - i - 1, & \text{if } 1 \leq i \leq n - 1 \\ 4n - 1, & \text{if } i = n \end{cases}$$

 ISBN 978-602-71141-0-4

$\lambda_1(\overrightarrow{x_i y_i}) = i, \quad \text{if } 1 \le i \le n$

Then we get the label vertices and arcs of sun digraph $\overrightarrow{S_n}$, that is both of $\lambda_1(A) = \{1, 2, \dots, n, 3n, \quad 3n+1, \dots, 4n-1\}$ and $\lambda_1(V) = \{n+1, n+2, \dots, 3n-1, 4n\}$ was a set which complementary. So it is easy to verify that the labeling λ_1 is a bijection form the set $V \cup A$ onto the set $\{1, 2, 3, \dots, 4n\}$.

Let us denote the weight of the vertices x_i and y_i of $\overrightarrow{S_n}$ under the labeling λ_1 by:

$$wt_{\lambda_1}(x_i) = \lambda_1(\overrightarrow{x_{i-1} x_i}) + \lambda_1(x_i) - \lambda_1(\overrightarrow{x_i x_{i+1}}) - \lambda_1(\overrightarrow{x_i y_i})$$

And

$$wt_{\lambda_1}(y_i) = \lambda_1(\overrightarrow{x_i y_i}) + \lambda_1(y_i)$$

Then for all $i = 1, 2, \dots, n$ and $i+1$ is taken modulo n, the weight of the vertices x_i can be determined as following way:

- for $1 \le i \le n-1$, we have
$$\begin{aligned} wt_{\lambda_1}(x_i) &= \lambda_1(\overrightarrow{x_{i-1} x_i}) + \lambda_1(x_i) - \lambda_1(\overrightarrow{x_i x_{i+1}}) - \lambda_1(\overrightarrow{x_i y_i}) \\ &= (4n - (i-1) - 1) + (2n + i) - (4n - i - 1) - (i) \\ &= 2n + 1 \end{aligned}$$
- for $i = n$, we have
$$\begin{aligned} wt_{\lambda_1}(x_n) &= \lambda_1(\overrightarrow{x_{n-1} x_n}) + \lambda_1(x_n) - \lambda_1(\overrightarrow{x_n x_1}) - \lambda_1(\overrightarrow{x_n y_n}) \\ &= (4n - (n-1) - 1) + 4n - (4n-1) - (n) \\ &= 2n + 1 \end{aligned}$$

By the similar way, for all $i = 1, 2, \dots, n$ and $i+1$ is taken modulo n, the weight of the vertices y_i can be determined as following way:

$$\begin{aligned} wt_{\lambda_1}(y_i) &= \lambda_1(\overrightarrow{x_i y_i}) + \lambda_1(y_i) \\ &= i + (2n + 1 - i) \\ &= 2n + 1 \end{aligned}$$

Since the weight of vertices x_i and y_i is $wt_{\lambda_1}(x_i) = wt_{\lambda_1}(y_i) = 2n+1$, for all $1 \le i \le n$, then λ_1 is a vertex magic total labeling with magic constant $2n+1$. Thus, sun digraph $\overrightarrow{S_n}$, $n \ge 3$, has a vertex magic total labeling with magic constant $2n+1$ is proved. □

The example of vertex magic total labeling on sun digraph $\overrightarrow{S_n}$ is given by Figure 1.

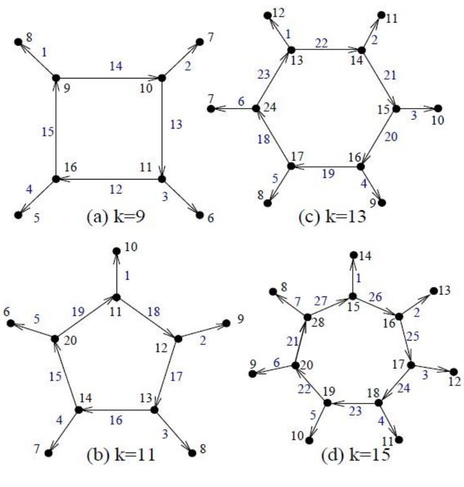

Figure 1. Vertex Magic Total Labeling on (a) $\overrightarrow{S_4}$, (b) $\overrightarrow{S_5}$, (c) $\overrightarrow{S_6}$, and (d) $\overrightarrow{S_7}$

Vertex Magic Total Labeling on Disjoint union of Sun Digraphs

The disjoint union of *m copies* sun digraphs $\overrightarrow{S_n}$, denoted by $m\overrightarrow{S_n}$, is defined as digraph with vertex set $V = \{x_i^c | 1 \le i \le n, 1 \le j \le m\} \cup \{y_i^j | 1 \le i \le n, 1 \le j \le m\}$ and arc set $A = \{\overrightarrow{x_i^j x_{i+1}^j} | 1 \le i \le n, 1 \le$

$j \leq m\} \cup \{\overrightarrow{x_i^j y_i^j} | 1 \leq i \leq n,\ 1 \leq j \leq m\}$, with $n \geq 3$ and $i+1$ is taken modulo *n*. The order and size of $m\overrightarrow{S_n}$ respectively are $2nm$ vertices and $2nm$ arcs. The following theorem describes a vertex magic total labeling on digraph $m\overrightarrow{S_n}$, for $m \geq 2$ and $n \geq 3$.

Theorem 2. *For* $m \geq 2$ and $n \geq 3$, *the m-copies of sun digraphs* $m\overrightarrow{S_n}$ *has a vertex magic total labeling with magic constant* $2mn+1$.

Proof. Let *s* and *t* are order and size of $m\overrightarrow{S_n}$, thus $s = 2mn$ and $t = 2mn$. Define total labeling $\lambda_2: V \cup A \longrightarrow (1, 2, 3, \ldots, s+t)$ in the following way:

- for $1 \leq i \leq n-1$, we have
$$\lambda_2(x_i^j) = (2n+i-1)m + j, \quad 1 \leq j \leq m$$

$$\lambda_2\left(\overrightarrow{x_i^j x_{i+1}^j}\right) = \begin{cases} 3mn + n - (m+1) - (i-1), & \text{j} = 1 \\ (3m+j)n - 1, & 2 \leq j \leq m \end{cases}$$

- for $i = n$, we have
$$\lambda_2(x_i^j) = 3mn - m + n + j, \quad i = n$$

$$\lambda_2\left(\overrightarrow{x_i^j x_{i+1}^j}\right) = \begin{cases} 3mn - m + n, & \text{j} = 1 \\ (3m+j)n, & 2 \leq j \leq m \end{cases}$$

- for $1 \leq i \leq n$ and $1 \leq j \leq m$, we have
$$\lambda_2(y_i^j) = (2n+1-i)m - (j-)$$

$$\lambda_2\left(\overrightarrow{x_i^j y_i^j}\right) = (i-1)m + j$$

Then we get the label vertices and arcs of $m\overrightarrow{S_n}$, that is:

$$\lambda_2(A) = \{1, 2, .., mn, (3n-1)m + 1, (3n-1)m + 2, \ldots, (3n-1) + n, (3m+1)n + 1, (3m+1)n + 2, \ldots, 4mn\}$$

and

$$\lambda_2(V) = \{mn+1, mn+2, \ldots, (3n-1)m, (3n-1)m + n + 1, (3n-1)m + n + 2, \ldots, (3m+1)n\}$$

where $\lambda_2(A)$ and $\lambda_2(V)$ were a set which complementary. So it is easy to verify that the labeling λ_2 is a bijection form the set $V \cup A$ onto the set $\{1, 2, \ldots, 4mn\}$.

Let us denote the weight of the vertices x_i^j and y_i^j of $m\overrightarrow{S_i}$ under the labeling λ_2 by:

$$wt_{\lambda_2}(x_i^j) = \lambda_2\left(\overrightarrow{x_{i-1}^j x_i^j}\right) + \lambda_2(x_i^j) - \lambda_2\left(\overrightarrow{x_i^j x_{i+1}^j}\right) - \lambda_2\left(\overrightarrow{x_i^j y_i^j}\right)$$ and

$$wt_{\lambda_2}(y_i^j) = \lambda_2\left(\overrightarrow{x_i^j y_i^j}\right) + \lambda_2(y_i^j)$$

Then, for all $j = 1, 2, \ldots, m$, $i = 1, 2, \ldots, n$ and $i+1$ is taken modulo *n*, the weight of the vertices x_i^j can be determined as following way:

- for $j = 1$ and $1 \leq i \leq n-1$, we have
$$\begin{aligned} wt_{\lambda_2}(x_i^1) &= \lambda_2\left(\overrightarrow{x_{i-1}^1 x_i^1}\right) + \lambda_2(x_i^1) - \lambda_2\left(\overrightarrow{x_i^1 x_{i+1}^1}\right) - \lambda_2(x_i^1 y_i^1) \\ &= (3mn + n - (m+1) - ((i-1) - 1)) + ((2n+i-1)m + 1) - (3mn + n - (m+1) - (i-1)) - ((i-1)m + 1) \\ &= 2mn + 1 \end{aligned}$$

- for $j = 1$ and $i = n$, we have
$$\begin{aligned} wt_{\lambda_2}(x_n^1) &= \lambda_2\left(\overrightarrow{x_{n-1}^1 x_i^1}\right) + \lambda_2(x_n^1) - \lambda_2\left(\overrightarrow{x_n^1 x_1^1}\right) - \lambda_2(x_n^1 y_n^1) \\ &= (3mn + n - (m+1) - ((n-1) - 1)) + (3mn - m + n + 1) - (3mn - m + n) - ((n-1)m + 1) \\ &= 2mn + 1 \end{aligned}$$

ISBN 978-602-71141-0-4

- for $2 \le j \le m$ and $1 \le i \le n-1$, we have

$$wt_{\lambda_2}\left(x_i^j\right) = \lambda_2\left(\overrightarrow{x_{i-1}^j x_i^j}\right) + \lambda_2\left(x_i^j\right) - \lambda_2\left(\overrightarrow{x_i^j x_{i+1}^j}\right) - \lambda_2\left(\overrightarrow{x_i^j y_i^j}\right)$$
$$= \big((3m+j)n - (i-1)\big) + ((2n + i - 1)m + j - \big((3m + j)n - i\big) - ((i-1)m + j)$$
$$= 2mn + 1$$

- for $2 \le j \le m$ and $i = n$, we have

$$wt_{\lambda_2}\left(x_n^j\right) = \lambda_2\left(\overrightarrow{x_{n-1}^j x_i^j}\right) + \lambda_2\left(x_n^j\right) - \lambda_2\left(\overrightarrow{x_n^j x_1^j}\right) - \lambda_2\left(x_n^j y_n^j\right)$$
$$= \big((3m+j)n - (n-1)\big) - (3mn - m + n + j) + ((3m + j)n - \big((n-1)m + j\big)$$
$$= 2mn + 1$$

By the similar way, for all $j = 1,2,\ldots,m$, $i = 1,2,\ldots,n$ and $i+1$ is taken modulo n, the weight of the vertices y_i^j can be determined as following way:

$$wt_{\lambda_2}\left(y_i^j\right) = \lambda_2\left(\overrightarrow{x_i^j y_i^j}\right) + \lambda_2\left(y_i^j\right)$$
$$= \big((i-1)m + j\big) - ((2n - i + 1)m - (j-1))$$
$$= 2mn + 1$$

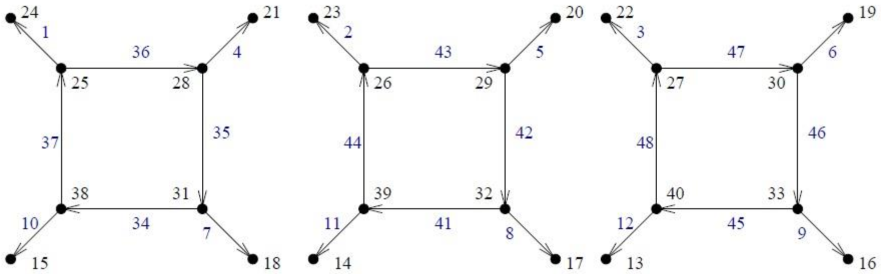

Figure 2. Vertex Magic Total Labeling on $3\overrightarrow{S_4}$ with $k = 25$

Since the weight of vertices x_i^j and y_i^j is $wt_{\lambda_2}\left(x_i^j\right) = wt_{\lambda_2}\left(y_i^j\right) = 2mn + 1$ for all $1 \le j \le m$ and $1 \le i \le n$, then λ_2 is a vertex magic total labeling with magic constant $2mn + 1$. Thus, digraph $m\overrightarrow{S_n}$, $m \ge 2$ and $n \ge 3$, has a vertex magic total labeling with magic constant $2mn + 1$ is proved. □

The example of vertex magic total labeling on disjoint union of 3 *copies* of $\overrightarrow{S_4}$ is given in Figure 2. We note that if $m = 1$, then $m\overrightarrow{S_n}$ is isomorphic to $\overrightarrow{S_n}$. In this case Theorem 2 shows the vertex magic total labeling on $\overrightarrow{S_n}$ with magic constant $2mn + 1 = 2(1)n + 1 = 2n + 1$. In general we can combine Theorem 1 and Theorem 2 as follow:

Theorem 2'. *For* $m \in \mathbb{N}$ and $n \ge 3$, *the m-copies of sun digraphs* $m\overrightarrow{S_n}$ *has a vertex magic total labeling with magic constant* $2mn + 1$.

Super Vertex Magic Total Labeling on Sun Digraphs

In this section we consider a super vertex magic total labeling on sun digraph. Vertex magic total labeling on *G* is called *super* if $\lambda(V) = \{1,2,\ldots,p\}$ (MacDougall *et al.*, 2004). Thus, vertex magic total labeling on digraph *D* is called *super* if $\lambda: V \longrightarrow \{1,2,\ldots,s\}$ and $\lambda: A \longrightarrow \{s+1,\ldots,s+t\}$, for $s = |V|$ and $t = |A|$. The following theorem present super vertex magic total labeling of *m-copies* of sun digraph.

Theorem 3. *The m-copies of sun digraphs* $m\overrightarrow{S_n}$ *is not super vertex magic total labeling for all* $m \in \mathbb{N}$ *and* $n \ge 3$.

Proof. Let *s* and *t* are order and size of $m\overrightarrow{S_n}$ respectively, thus $s = 2mn$ and $t = 2mn$.

ISBN 978-602-71141-0-4

Define λ_3 is one-to-one map $\lambda_3: V \longrightarrow \{1,2,\ldots,s\}$ and $\lambda_3: A \longrightarrow \{s+1, s+2,\ldots,s+t\}$. If $Wt_{ë_3}$ is sum of the weight of vertices, $wt_{ë_3}$, from all vertices x_i^j and y_i^j in $m\overrightarrow{S_n}$, then $Wt_{ë_3}$ is given by following way:

$$Wt_{ë_3} = \sum_{f=s+1}^{s+t} f + \sum_{g=1}^{s} g - \sum_{f=s+1}^{s+t} f$$
$$= \sum_{g=1}^{2mn} g$$
$$= \frac{2mn(2mn+1)}{2}$$

If k is magic constant for super vertex magic total labeling on $m\overrightarrow{S_n}$, then we get:

$$k = \overline{Wt_{ë_3}}$$
$$= \frac{\frac{2mn(2mn+1)}{2}}{2mn}$$
$$= \frac{2mn+1}{2}$$

for all $m \in \mathbb{N}$ *and* $n \geq 3$, we get magic constant k is not integer. This condition is contradiction that k is require equal to the weight of vertices, so that magic constant k must be integer. Thus, there is no super vertex magic total labeling on digraphs $m\overrightarrow{S_n}$ for all $m \in \mathbb{N}$ and $n \geq 3$. □

Since the order and size of $m\overrightarrow{S_n}$ is $s = t = 2mn$, then we called vertex magic total labeling is *anti-super* if $ë: V \longrightarrow \{s+1, s+2,\ldots,s+t\}$.

Theorem 3'. *There is no anti-super vertex magic total labeling on the m-copies of sun digraphs $m\overrightarrow{S_n}$ for all $m \in \mathbb{N}$ and $n \geq 3$.*

Proof. Define $ë_4$ is anti-super vertex magic total labeling on sun digraphs $m\overrightarrow{S_n}$, then $ë_4: V \longrightarrow \{s+1, s+2,\ldots,s+t\}$ and $ë_4: A \longrightarrow \{1,2,\ldots,s\}$. Let $Wt_{ë_4}$ is sum of the vertices weight, $wt_{ë_4}$, from all vertices x_i^j and y_i^j in $m\overrightarrow{S_n}$, then $Wt_{ë_4}$ is given by following way:

$$Wt_{ë_4} = \sum_{f=1}^{s} f + \sum_{g=s+1}^{s+t} g - \sum_{f=1}^{s} f$$
$$= \sum_{g=2mn+1}^{4mn} g$$
$$= \frac{2mn(4mn+2mn+1)}{2}$$
$$= \frac{2mn(6mn+1)}{2}$$

If k is magic constant for anti-super vertex magic total labeling on $m\overrightarrow{S_n}$, then we get:

$$k = \overline{Wt_{ë_4}}$$
$$= \frac{\frac{2mn(6mn+1)}{2}}{2mn}$$
$$= \frac{6mn+1}{2}$$

for all $m \in \mathbb{N}$ *and* $n \geq 3$, we get k is not integer. This condition is contradiction that k is require equal to the weight of vertices, so magic constan k must be integer. Therefore, there is no anti-super vertex magic total labeling on digraphs $m\overrightarrow{S_n}$ for all $m \in \mathbb{N}$ and $n \geq 3$. □

CONCLUSION

We conclude this paper with the following open problem related to vertex magic total labeling on sun digraph.

Open problem 1. *Find vertex magic total labeling on disjoint union of non-isomorphic sun digraph.*

REFERENCES

D. Hefetz, T. Mütze, J. Schwartz. 2010. On antimagic directed graphs, *Journal of Graph Theory*, **64**, Issue 3, 219–232.

Dafik, 2008. *Handout: Pemodelan Matematika*, Jember: FKIP Universitas Jember.

J. A. Gallian. 2013. A Dynamic Survey of Graph Labelling, *The Electronic Journal of Combinatorics*, **16,** # DS6.

J.A. MacDougall, M. Miller and K.A. Sugeng. 2004. Super Vertex-magic total labelings of Graphs, *Proceedings of the 15th Australasian Workshop on Combinatorial Algorithms*, 222-229.

J.A. MacDougall, M. Miller, Slamin and W.D. Wallis, 2002. Vertex-magic

 ISBN 978-602-71141-0-4

total labeling, *Utilitas Math*, **61**, 3-21.

M.T. Rahim and Slamin , 2012. On vertex-magic total labeling of union of sun graphs, *Ars Combinatoria*, **103**, 305-310.

N. Hartsfield and G. Ringel, 1994. *Pearls in Graph Theory*, Academic Press, San Diego.

Slamin, 2009. *Desain Jaringan: Pendekatan Teori Graf*, Jember: Jember University Press.

Unit of Publication
Department of Mathematics Education
Faculty of Teacher Training and Education
Islamic University of Malang (UNISMA)
Malang, East Java, Indonesia

ISBN 978-602-71141-0-4