ISBN : 978-602- 19240-0-6

PROSIDING

SEMINAR NASIONAL

MATEMATIKA &

PENDIDIKAN MATEMATIKA

Tema :

"Peran Matematika dan Pembelajarannya dalam Meningkatkan Daya Saing Bangsa"

PROGRAM STUDI PENDIDIKAN MATEMATIKA
FKIP UNIVERSITAS JEMBER
2011

ISBN : 978-602- 19240-0-6

PROSIDING

SEMINAR NASIONAL

MATEMATIKA &

PENDIDIKAN MATEMATIKA

Tema :

"Peran Matematika dan Pembelajarannya dalam Meningkatkan Daya Saing Bangsa"

PROGRAM STUDI PENDIDIKAN MATEMATIKA
FKIP UNIVERSITAS JEMBER
2011

Susunan Tim Penyunting

Program Studi Pendidikan Matematika
FKIP Universitas Jember
Jl. Kalimantan 37 Tegalboto Jember, 68121

DAFTAR ISI

Makalah Utama

Makalah Matematika Murni

Makalah Pendidikan Matematika

Makalah Matematika Murni (2)

Makalah Pendidikan Matematika (2)

Super Edge Antimagic Total Labeling on Disjoint Union of Cycle Non Isomorphic

Yuni Listiana[1], Dafik[1], Slamin[2]

[1]Department of Mathematics Education, University of Jember
{yuni.listiana7, d.dafik}@gmail.com

[2]Department of System Information, University of Jember
slamins@gmail.com

Abstract

A graph G with order p and size q is called (a,d)-edge antimagic total ((a,d)-EAT) if there exist integers $a > 0$, $d \geq 0$ and a bijection $\alpha : V \cup E \rightarrow \{1,2,3,...,p+q\}$ such that $W = \{w(uv), uv \in E\} = \{a, a+d, ..., a+(q-1)d\}$, where $w(uv) = \alpha(u) + \alpha(v) + \alpha(uv)$. An (a,d)-EAT labeling α of graph G is super if $\alpha(V) = \{1,2,...,p\}$. In this paper we describe how to construct a super (a,d)-EAT labeling on some classes of disjoint union from non isomorphic graphs, namely disjoint union of cycle non isomorphic, $C_n \cup C_{n+2} \cup C_{n+4}$.

Keywords : *(a,d)-edge-antimagic total labeling, super(a,d)-edge antimagic total labelling, $C_n \cup C_{n+2} \cup C_{n+4}$.*

1 Introduction

All graphs in this paper are finite, undirected, and simple. $V(G)$ and $E(G)$ (in short, V and E) stand for the vertex-set and edge-set of graph G, respectively. Let $e = \{u,v\}$ (in short, $e = uv$) denote an edge connecting vertices u and v in G. Then, let order $|V(G)|$ in G denoted by p and size $|E(G)|$ in G denoted by q.

A *labeling* of a graph is any mapping that sends some set of graph elements to a set of numbers (usually to the positive integers). If the domain is the vertex-set or the edge-set, the labelings are called respectively *vertex-labelings* or *edge-labelings.* In this paper we deal with the case where the domain is $V \cup E$, and these are called *total-labelings.* We define the *edge-weight* of an edge $uv \in E$ under a total labeling to be the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv. General references for graph-theoretic notions is [12]. A general survey of graph labelings is [6].

A graph G is called (a,d)*-edge antimagic total* ($(a,d) - EAT$) if there exist integers $a > 0$, $d \geq 0$ and a bijection function $f : V \cup E \rightarrow \{1,2,\ldots,p+q\}$ such that the set of edge-weights is $w(uv) = f(u) + f(uv) + f(v), uv \in E$, form an arithmetic progression $\{a, a+d, a+2d, \ldots, a+(q-1)d\}$. In particular, an $(a,d) - EAT$ labeling of graph G is super if $f : V \rightarrow \{1,2,\ldots,p\}$. Thus, a *super* (a,d)*-edge-antimagic total graph* is a graph that admits a (a,d)-edge-antimagic total labeling.

The concept of (a,d)-edge antimagic total labeling, introduced by Simanjuntak *et al.* in [14], is natural extension of the notion of *edge-magic* labeling defined by Kotzig and

Rosa [1] (see also [9], [13], [3] and [15]). The super (a,d)-edge-antimagic total labeling is natural extension of the notion of *super edge-magic* labeling which was defined by Enomoto *et. al.* in [4].

In this paper we investigate the existence of super (a,d)-edge-antimagic total labelings for disjoint union of non isomorphic graphs. A number of classification studies on super $(a,d)-EAT$ (resp. $(a,d)-EAT$) for disjoint union of non isomorphic graphs has been extensively investigated. For instances, some constructions of super $(a,0)$-edge-antimagic total labelings for $nC_k \cup mP_k$ and $K_{1,m} \cup K_{1,n}$ have been shown by Ivančo and Lučkaničová in [7]. In [5] Sudarsana, Ismaimuza, Baskoro, and Assiyatun show that $P_n \cup P_{n+1}$, $nP_2 \cup P_n$ $(n \geq 2)$, and $nP_2 \cup P_{n+2}$ are super edge antimagic total labeling. Dafik *et al* also found disjoint union of non isomorphic graph which admits super (a,d)-edge-antimagic total labelings, namely $mK_{1,m} \cup S_{k,1}$ in [2].

More results concerning on super edge antimagic total labeling, see for instances in a nice survey paper by Gallian [6].

Now, we will concentrate on the disjoint union of cycle non isomorphic, denoted by $C_n \cup C_{n+2} \cup C_{n+4}$, for $n \geq 3$ and n odd.

2 Some Useful Lemmas

We start this section by a necessary condition for a graph to be super (a,d)-edge-antimagic total, providing a least upper bound for feasible values of d.

Lemma 1 *If a (p,q)-graph is super (a,d)-edge-antimagic total then $d \leq \frac{2p+q-5}{q-1}$.*

Proof. Assume that a (p,q)-graph has a super (a,d)-edge-antimagic total labeling $f: V(G) \cup E(G) \to \{1,2,\ldots,p+q\}$. The minimum possible edge-weight in the labeling f is at least $1+2+p+1 = p+4$. Thus, $a \geq p+4$. On the other hand, the maximum possible edge-weight is at most $(p-1)+p+(p+q) = 3p+q-1$. So we obtain $a+(q-1)d \leq 3p+q-1$ which gives the desired upper bound for the difference d. □

The following lemma, proved by Figueroa-Centeno *et al.* in [13], gives a necessary and sufficient condition for a graph to be super edge magic (super $(a,0)$-edge antimagic total).

Lemma 2 *A (p,q)-graph G is super edge-magic if and only if there exists a bijective function $f: V(G) \to \{1,2,\ldots,p\}$ such that the set $S = \{f(u)+f(v) : uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with magic constant $a = p+q+s$, where $s = min(S)$ and $S = \{a-(p+1), a-(p+2), \ldots, a-(p+q)\}$.*

In our terminology, the previous lemma states that a (p,q)-graph G is super $(a,0)$-edge antimagic total if and only if there exists an $(a-p-q,1)$-edge antimagic vertex labeling.

Next, we restate the following lemma that appeared in [8].

Lemma 3 [8] *Let $\mathfrak{A}$ be a sequence $\mathfrak{A} = \{c, c+1, c+2, \ldots c+k\}$, k even. Then there exists a permutation $\Pi(\mathfrak{A})$ of the elements of $\mathfrak{A}$ such that $\mathfrak{A} + \Pi(\mathfrak{A}) = \{2c + \frac{k}{2}, 2c + \frac{k}{2} + 1, 2c + \frac{k}{2} + 2, \ldots, 2c + \frac{3k}{2} - 1, 2c + \frac{3k}{2}\}$.*

3 Research Methods and Techniques

Research methods a super (a, d)-edge-antimagic total labeling on disjoint union of cycle non isomorphic are deductive axiomatic and the pattern recognition. Then for research techniques are as follows:

1. calculate the order p and size q on disjoint union of cycle non isomorphic;
2. determine the upper bound for values of d on disjoint union of cycle non isomorphic according by Lemma 1;
3. find label $EAVL$ or edge antimagic vertex labeling on disjoint union of cycle non isomorphic;
4. if the label of $EAVL$ is *expandable*, then we continue to determine the bijective function of $EAVL$;
5. find label $SEATL$ on disjoint union of cycle non isomorphic with feasible values of d,
6. determine the bijective function of super-edge antimagic total labeling on disjoint union of cycle non isomorphic.

4 Disjoint Union of Cycle Non Isomorphic

Disjoint union of Cycle Non Isomorphic denoted by $C_n \cup C_{n+2} \cup C_{n+4}$, $n \geq 3$ and n odd, is a disconnected graph with vertex set $V = \{x_i^1 \mid 1 \leq i \leq n\} \cup \{x_j^2 \mid 1 \leq j \leq n+2\} \cup \{x_k^3 \mid 1 \leq k \leq n+4\}$, and edge set:
$E = \{x_i^1 x_{i+1}^1 \cup x_n^1 x_1^1 \mid 1 \leq i \leq n-1\} \cup \{x_j^2 x_{j+1}^2 \cup x_{n+2}^2 x_1^2 \mid 1 \leq i \leq n+1\}$
$\cup \{x_k^3 x_{k+1}^3 \cup x_{n+4}^3 x_1^3 \mid 1 \leq k \leq n+3\}$.
Thus, $p = q = n + (n+2) + (n+4) = 3n + 6$.

If the disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$, has a super (a, d)-edge antimagic total labeling then, for $p = q = 3n + 6$, it follows from Lemma 1 that the upper bound of d is $d \leq 3 - \frac{2}{3n+5}$, $d \geq 0$, d is integer, so $d \in \{0, 1, 2\}$

The following theorem describes an $(a, 1)$-edge antimagic vertex labeling on disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$, $n \geq 3$ and n odd.

Lemma 4 *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has an $(\frac{3n+9}{2}, 1)$-edge antimagic vertex labeling for $n \geq 3$ and n odd.*

Proof. Define the vertex labeling $\alpha_1 : V(C_n \cup C_{n+2} \cup C_{n+4}) \to \{1, 2, \ldots, 3n+6\}$ in the following way:

For x_i^1, $\quad 1 \le i \le n$

$$\alpha_1(x_i^1) = \frac{3i+3}{2} + \left(\frac{(-1)^i+1}{4}\right) 3n$$

For x_j^2, $\quad 1 \le j \le n+2$

$$\alpha_1(x_j^2) = \begin{cases} \frac{3j+1}{2} + \left(\frac{(-1)^j+1}{4}\right) 3n + \left(\frac{(-1)^j+1}{2}\right) 3, & \text{if } 1 \le j \le n \\ \frac{1}{2} + \left(\frac{(-1)^j+1}{2}\right) 3n + \left(\frac{(-1)^{j+1}+1}{4}\right) 3j + \left(\frac{(-1)^j+1}{4}\right) 11, & \text{if } n+1 \le j \le n+2 \end{cases}$$

For x_k^3, $\quad 1 \le k \le n+4$

$$\alpha_1(x_k^3) = \begin{cases} \frac{3k-1}{2} + \left(\frac{(-1)^k+1}{4}\right) 3n + \left(\frac{(-1)^k+1}{2}\right) 6, & \text{if } 1 \le k \le n \\ \frac{3k-1}{2} + \left(\frac{(-1)^k+1}{2}\right) 4n - \left(\frac{(-1)^k+1}{4}\right) 5k + \left(\frac{(-1)^k+1}{4}\right) 13, & \text{if } n+1 \le k \le n+4 \end{cases}$$

The vertex labeling α_1 is a bijective function. The edge weights of $C_n \cup C_{n+2} \cup C_{n+4}$, under the labeling α_1, constitute the following sets:

For i, $\quad 1 \le i \le n$

$$\begin{aligned} w_{\alpha_1}^1(x_i^1 x_{i+1}^1) &= \tfrac{3n+6i+9}{2}, && \text{if } 1 \le i \le n-1 \\ w_{\alpha_1}^2(x_n^1 x_1^1) &= \tfrac{3n+9}{2} \end{aligned}$$

For j, $\quad 1 \le j \le n+2$

$$\begin{aligned} w_{\alpha_1}^3(x_j^2 x_{j+1}^2) &= \tfrac{3n+6j+11}{2}, && \text{if } 1 \le j \le n-1 \\ w_{\alpha_1}^4(x_j^2 x_{j+1}^2) &= \tfrac{3n+6j+13}{2}, && \text{if } n \le j \le n+1 \\ w_{\alpha_1}^5(x_{n+2}^2 x_1^2) &= \tfrac{3n+11}{2} \end{aligned}$$

For k, $\quad 1 \le k \le n+4$

$$\begin{aligned} w_{\alpha_1}^6(x_k^3 x_{k+1}^3) &= \tfrac{3n+6k+13}{2}, && \text{if } 1 \le k \le n-1 \\ w_{\alpha_1}^7(x_k^3 x_{k+1}^3) &= \tfrac{3n+6k+9}{2}, && \text{if } n \le k \le n+1 \\ w_{\alpha_1}^8(x_k^3 x_{k+1}^3) &= \tfrac{3n+6k-1}{2}, && \text{if } n+2 \le k \le n+3 \\ w_{\alpha_1}^9(x_{n+4}^3 x_1^3) &= \tfrac{3n+13}{2} \end{aligned}$$

It is not difficult to see that the set of

$$\bigcup_{t=1}^{9} W_{\alpha_1}^t = \left\{\tfrac{3n+9}{2}, \tfrac{3n+11}{2}, \tfrac{3n+13}{2} \ldots, \tfrac{9n+19}{2}\right\},$$

consists of consecutive integers. Thus α_1 is a $\left(\frac{3n+9}{2}, 1\right)$-edge antimagic vertex labeling. □

Bača, Y. Lin, M. Miller and R. Simanjuntak (See [9], Theorem 5) have proved that if (p,q)-graph G has an (a,d)-edge antimagic vertex labeling then G has a super$(a+p+q, d-1)$-edge antimagic total labeling and a super$(a+p+1, d+1)$-edge antimagic total labeling. With the theorem 1 in hand, and using Theorem 5 from [9], we obtain the following result.

Theorem 1 *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $\left(\frac{15n+33}{2}, 0\right)$-edge antimagic total labeling and a super $\left(\frac{9n+23}{2}, 2\right)$-edge antimagic total labeling, for $n \geq 3$ and n odd.*

Proof. *Case 1.* for $d = 0$
We have proved that the vertex labeling α_1 is a $\left(\frac{3n+9}{2}, 1\right)$-edge antimagic vertex labeling. With respect to Lemma 2, by completing the edge labels $p+1, p+2, \ldots, p+q$, we are able to extend labeling α_1 to a super $(a, 0)$-edge antimagic total labeling, where, for $p = q = 3n + 6$, the value $a = \frac{15n+33}{2}$ and and common difference 0. Thus a super $\left(\frac{15n+33}{2}, 0\right)$-edge-antimagic total labeling. This concludes the proof. □

Proof. *Case 2.* for $d = 2$
Label the vertices of $C_n \cup C_{n+2} \cup C_{n+4}$ with $\alpha_2(x_i^1) = \alpha_1(x_i^1)$, $\alpha_2(x_j^2) = \alpha_1(x_j^2)$ and $\alpha_2(x_k^3) = \alpha_1(x_k^3)$, for $i = 1, 2, ..., n$, $j = 1, 2, ..., n+2$ and $k = 1, 2, ..., n+4$; and label the edges with the following way.

For i, $\quad 1 \leq i \leq n$

$$\begin{aligned} \alpha_2(x_i^1 x_{i+1}^1) &= 3n + 3i + 7, \quad \text{if } 1 \leq i \leq n-1 \\ \alpha_2(x_n^1 x_1^1) &= 3n + 7 \end{aligned}$$

For j, $\quad 1 \leq j \leq n+2$

$$\begin{aligned} \alpha_2(x_j^2 x_{j+1}^2) &= 3n + 3j + 8, \quad \text{if } 1 \leq j \leq n-1 \\ \alpha_2(x_j^2 x_{j+1}^2) &= 3n + 3j + 9, \quad \text{if } n \leq j \leq n+1 \\ \alpha_2(x_{n+2}^2 x_1^2) &= 3n + 8 \end{aligned}$$

For k, $\quad 1 \leq k \leq n+4$

$$\begin{aligned} \alpha_2(x_k^3 x_{k+1}^3) &= 3n + 3k + 9, \quad \text{if } 1 \leq k \leq n-1 \\ \alpha_2(x_k^3 x_{k+1}^3) &= 3n + 3k + 7, \quad \text{if } n \leq k \leq n+1 \\ \alpha_2(x_k^3 x_{k+1}^3) &= 3n + 3k + 2, \quad \text{if } n+2 \leq k \leq n+3 \\ \alpha_2(x_{n+4}^3 x_1^3) &= 3n + 9 \end{aligned}$$

The total labeling α_2 is a bijective function from $V(C_n \cup C_{n+2} \cup C_{n+4}) \cup E(C_n \cup C_{n+2} \cup C_{n+4})$ onto the set $\{1, 2, 3, \ldots, 6n+12\}$. The edge-weights of $C_n \cup C_{n+2} \cup C_{n+4}$, under the labeling α_2, constitute the sets:

For i, $\quad 1 \leq i \leq n$

$$\begin{aligned} W_{\alpha_2}^1 &= \{w_{\alpha_1}^1 + \alpha_2(x_i^1 x_{i+1}^1), \text{ if } 1 \leq i \leq n-1\} \\ &= \left(\tfrac{3n+6i+9}{2}\right) + (3n + 3i + 7) \\ &= \tfrac{9n+12i+23}{2} \\ W_{\alpha_2}^2 &= \{w_{\alpha_1}^2 + \alpha_2(x_n^1 x_1^1)\} \\ &= \left(\tfrac{3n+9}{2}\right) + (3n + 7) \\ &= \tfrac{19n+23}{2} \end{aligned}$$

For j, $\quad 1 \leq j \leq n+2$

$$\begin{aligned}
W^3_{\alpha_2} &= \{w^3_{\alpha_1} + \alpha_2(x^2_j x^2_{j+1});\text{ if } 1 \leq j \leq n-1\} \\
&= (\tfrac{3n+6j+11}{2}) + (3n+3j+8) \\
&= \tfrac{9n+12j+27}{2} \\
W^4_{\alpha_2} &= \{w^4_{\alpha_1} + \alpha_2(x^2_j x^2_{j+1});\text{ if } n \leq j \leq n+1\} \\
&= (\tfrac{3n+6j+13}{2}) + (3n+3j+9) \\
&= \tfrac{9n+12j+31}{2} \\
W^5_{\alpha_2} &= \{w^5_{\alpha_1} + \alpha_2(x^2_{n+2} x^2_1) \\
&= (\tfrac{3n+11}{2}) + (3n+8) \\
&= \tfrac{9n+27}{2}
\end{aligned}$$

For k, $\quad 1 \leq k \leq n+4$

$$\begin{aligned}
W^6_{\alpha_2} &= \{w^6_{\alpha_1} + \alpha_2(x^3_k x^3_{k+1});\text{ if } 1 \leq k \leq n-1\} \\
&= (\tfrac{3n+6k+13}{2}) + (3n+3k+9) \\
&= \tfrac{9n+12k+31}{2} \\
W^7_{\alpha_2} &= \{w^7_{\alpha_1} + \alpha_2(x^3_k x^3_{k+1});\text{ if } n \leq k \leq n+1\} \\
&= (\tfrac{3n+6k+9}{2}) + (3n+3k+7) \\
&= \tfrac{9n+12k+23}{2} \\
W^8_{\alpha_2} &= \{w^8_{\alpha_1} + \alpha_2(x^3_k x^3_{k+1});\text{ jika } n+2 \leq k \leq n+3\} \\
&= (\tfrac{3n+6k-1}{2}) + (3n+3k+2) \\
&= \tfrac{9n+12k+3}{2} \\
W^9_{\alpha_2} &= \{w^9_{\alpha_1} + \alpha_2(x^3_{n+4} x^3_1) \\
&= (\tfrac{3n+13}{2}) + (3n+9) \\
&= \tfrac{9n+31}{2}
\end{aligned}$$

It is not difficult to see that the set of

$$\bigcup_{t=1}^{9} W^t_{\alpha_2} = \left\{\tfrac{9n+23}{2}, \tfrac{9n+27}{2}, \tfrac{9n+31}{2}, \ldots, \tfrac{21n+43}{2}\right\},$$

contains an arithmetic sequence with the first term $\frac{9n+23}{2}$ and common difference 2. Thus α_2 is a super $(\frac{9n+23}{2}, 2)$-edge antimagic total labeling. This concludes the proof. □

Theorem 2 *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $(6n+14, 1)$-edge antimagic total labeling for $n \geq 3$ and n odd.*

Proof. We will prove using Lemma 3. For $n \geq 3$ and n odd, consider the vertex labeling α_1 of the graph $C_n \cup C_{n+2} \cup C_{n+4}$ from Lemma 4 which is a $(\frac{3n+9}{2}, 1)$-EAV labeling. Let a sequence $\mathfrak{A} = \{c, c+1, c+2, \dots, c+k\}$ be the set of edge-weights of the vertex labeling α_1 for $c = \frac{3n+9}{2}$ and $k = 3n+5$. In Lemma 3, there exists a permutation $\Pi(\mathfrak{A})$ of the elements of $\mathfrak{A}$ such that $\mathfrak{A} + \left[\Pi(\mathfrak{A}) + \frac{k}{2}\right] = \{2c+k, 2c+k+1, \dots, 2c+2k\}$. If $\left[\Pi(\mathfrak{A}) + \frac{k}{2}\right]$ is an edge labeling of graph $C_n \cup C_{n+2} \cup C_{n+4}$,then $\mathfrak{A} + \left[\Pi(\mathfrak{A}) + \frac{k}{2}\right]$ gives the set of the edge weights of $C_n \cup C_{n+2} \cup C_{n+4}$, which implies that the resulting total labeling is super $(6n+14, 1)$. This concludes the proof. $\square$

5 Conclusion

We have lemma and theorem from bijective function of super (a, d)-edge antimagic total labeling on disjoint union of cycle non isomorphic:

- **Lemma 4** *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has an $(a, 1)$-edge antimagic vertex labeling for $n \geq 3$ and n odd.*
- **Theorem 1** *If $n \geq 3$ and n odd, the disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $\left(\frac{15n+33}{2}, 0\right)$-edge antimagic total labeling and a super $\left(\frac{9n+23}{2}, 2\right)$-edge-antimagic total labeling.*
- **Theorem 2** *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $(6n+14, 1)$-edge antimagic total labeling for $n \geq 3$ and n odd.*

References

[1] A. Kotzig and A. Rosa, Magic valuations of finite graphs, *Canad. Math. Bull.* **13** (1970), 451–461.

[2] Dafik, M. Miller, J. Ryan and M. Bača, On super (a, d)-edge antimagic total labeling of disconnected graphs, *Discrete Math.*, **309** (2009), 4909–4915.

[3] G. Ringel and A.S. Lladó, Another tree conjecture, *Bull. Inst. Combin. Appl.* **18** (1996), 83–85.

[4] H. Enomoto, A.S. Lladó, T. Nakamigawa and G. Ringel, Super edge-magic graphs, *SUT J. Math.* **34** (1998), 105–109.

[5] I.W. Sudarsana, D. Ismaimuza, E.T. Baskoro and H. Assiyatun, On super (a, d)-edge-antimagic total labeling of disconnected graphs, *JCMCC* **55** (2005), 149–158.

[6] J. A. Gallian, A Dynamic Survey of Graph Labelling, *Electronic Journal Combinatorics*, # DS6, (2010).

[7] J. Ivančo and I. Lučkaničová, On edge-magic disconnected graphs, *SUT Journal of Math.* **38** (2002), 175–184.

[8] K.A. Sugeng, M. Miller, Slamin and M. Bača, (a, d)-edge-antimagic total labelings of caterpillars, *Lecture Notes in Computer Science* 3330 (2005), 169–180.

[9] M. Bača, Y. Lin, M. Miller and R. Simanjuntak, New constructions of magic and antimagic graph labelings, *Utilitas Math.* **60** (2001), 229–239.

[10] M. Bača and M. Miller, *Super Edge Antimagic Graphs*, Brown Walker Press, Boca Raton, (2008).

[11] M. Bača, Dafik, M. Miller and J. Ryan, On super (a, d)-edge antimagic total labeling of caterpillars, *J. Combin. Math. Combin. Comput.*, **65** (2008), 61–70.

[12] N. Hartsfield and G. Ringel, *Pearls in Graph Theory*, Academic Press, San Diego, (1994).

[13] R.M. Figueroa-Centeno, R. Ichishima and F.A. Muntaner-Batle, The place of super edge-magic labelings among other classes of labelings, *Discrete Math.* **231** (2001), 153–168.

[14] R. Simanjuntak, F. Bertault and M. Miller, Two new (a, d)-antimagic graph labelings, *Proc. of Eleventh Australasian Workshop on Combinatorial Algorithms* (2000), 179–189.

[15] W. D. Wallis, E. T. Baskoro, M. Miller and Slamin , Edge-magic total labelings, *Austral. J. Combin.* **22** (2000), 177–190.