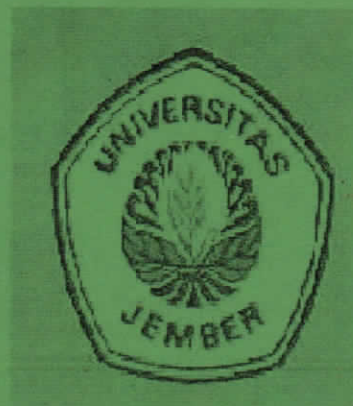


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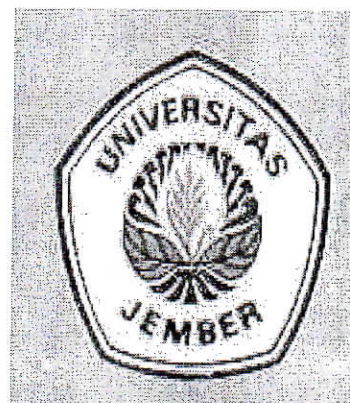
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Super Edge Antimagic Total Labeling on Disjoint Union of Cycle Non Isomorphic

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Abstract

A graph G with order p and size q is called (a, d) -edge antimagic total $((a, d)$ -EAT) if there exist integers $a > 0$, $d \geq 0$ and a bijection $\alpha : V \cup E \rightarrow \{1, 2, 3, \dots, p + q\}$ such that $W = \{w(uv), uv \in E\} = \{a, a + d, \dots, a + (q - 1)d\}$, where $w(uv) = \alpha(u) + \alpha(v) + \alpha(uv)$. An (a, d) -EAT labeling α of graph G is super if $\alpha(V) = \{1, 2, \dots, p\}$. In this paper we describe how to construct a super (a, d) -EAT labeling on some classes of disjoint union from non isomorphic graphs, namely disjoint union of cycle non isomorphic, $C_n \cup C_{n+2} \cup C_{n+4}$.

Keywords : (a, d) -edge-antimagic total labeling, super (a, d) -edge antimagic total labelling, $C_n \cup C_{n+2} \cup C_{n+4}$.

1 Introduction

All graphs in this paper are finite, undirected, and simple. $V(G)$ and $E(G)$ (in short, V and E) stand for the vertex-set and edge-set of graph G , respectively. Let $e = \{u, v\}$ (in short, $e = uv$) denote an edge connecting vertices u and v in G . Then, let order $|V(G)|$ in G denoted by p and size $|E(G)|$ in G denoted by q .

A *labeling* of a graph is any mapping that sends some set of graph elements to a set of numbers (usually to the positive integers). If the domain is the vertex-set or the edge-set, the labelings are called respectively *vertex-labelings* or *edge-labelings*. In this paper we deal with the case where the domain is $V \cup E$, and these are called *total-labelings*. We define the *edge-weight* of an edge $uv \in E$ under a total labeling to be the sum of the vertex labels corresponding to vertices u, v and edge label corresponding to edge uv . General references for graph-theoretic notions is [12]. A general survey of graph labelings is [6].

A graph G is called (a, d) -edge antimagic total $((a, d) - EAT)$ if there exist integers $a > 0$, $d \geq 0$ and a bijection function $f : V \cup E \rightarrow \{1, 2, \dots, p + q\}$ such that the set of edge-weights is $w(uv) = f(u) + f(v) + f(uv)$, $uv \in E$, form an arithmetic progression $\{a, a + d, a + 2d, \dots, a + (q - 1)d\}$. In particular, an $(a, d) - EAT$ labeling of graph G is super if $f : V \rightarrow \{1, 2, \dots, p\}$. Thus, a *super (a, d) -edge-antimagic total graph* is a graph that admits a (a, d) -edge-antimagic total labeling.

The concept of (a, d) -edge antimagic total labeling, introduced by Simanjuntak *et al.* in [14], is natural extension of the notion of *edge-magic* labeling defined by Kotzig and

Rosa [1] (see also [9], [13], [3] and [15]). The super (a, d) -edge-antimagic total labeling is natural extension of the notion of *super edge-magic* labeling which was defined by Enomoto *et al.* in [4].

In this paper we investigate the existence of super (a, d) -edge-antimagic total labelings for disjoint union of non isomorphic graphs. A number of classification studies on super $(a, d) - EAT$ (resp. $(a, d) - EAT$) for disjoint union of non isomorphic graphs has been extensively investigated. For instances, some constructions of super $(a, 0)$ -edge-antimagic total labelings for $nC_k \cup mP_k$ and $K_{1,m} \cup K_{1,n}$ have been shown by Ivančo and Lučkaničová in [7]. In [5] Sudarsana, Ismainuza, Baskoro, and Assiyatun show that $P_n \cup P_{n+1}$, $nP_2 \cup P_n$ ($n \geq 2$), and $nP_2 \cup P_{n+2}$ are super edge antimagic total labeling. Dafik *et al* also found disjoint union of non isomorphic graph which admits super (a, d) -edge-antimagic total labelings, namely $mK_{1,m} \cup S_{k,1}$ in [2].

More results concerning on super edge antimagic total labeling, see for instances in a nice survey paper by Gallian [6].

Now, we will concentrate on the disjoint union of cycle non isomorphic, denoted by $C_n \cup C_{n+2} \cup C_{n+4}$, for $n \geq 3$ and n odd.

2 Some Useful Lemmas

We start this section by a necessary condition for a graph to be super (a, d) -edge-antimagic total, providing a least upper bound for feasible values of d .

Lemma 1 *If a (p, q) -graph is super (a, d) -edge-antimagic total then $d \leq \frac{2p+q-5}{q-1}$.*

Proof. Assume that a (p, q) -graph has a super (a, d) -edge-antimagic total labeling $f : V(G) \cup E(G) \rightarrow \{1, 2, \dots, p+q\}$. The minimum possible edge-weight in the labeling f is at least $1 + 2 + p + 1 = p + 4$. Thus, $a \geq p + 4$. On the other hand, the maximum possible edge-weight is at most $(p - 1) + p + (p + q) = 3p + q - 1$. So we obtain $a + (q - 1)d \leq 3p + q - 1$ which gives the desired upper bound for the difference d . $\square$

The following lemma, proved by Figueroa-Centeno *et al.* in [13], gives a necessary and sufficient condition for a graph to be super edge magic (super $(a, 0)$ -edge antimagic total).

Lemma 2 *A (p, q) -graph G is super edge-magic if and only if there exists a bijective function $f : V(G) \rightarrow \{1, 2, \dots, p\}$ such that the set $S = \{f(u) + f(v) : uv \in E(G)\}$ consists of q consecutive integers. In such a case, f extends to a super edge-magic labeling of G with magic constant $a = p + q + s$, where $s = \min(S)$ and $S = \{a - (p + 1), a - (p + 2), \dots, a - (p + q)\}$.*

In our terminology, the previous lemma states that a (p, q) -graph G is super $(a, 0)$ -edge antimagic total if and only if there exists an $(a - p - q, 1)$ -edge antimagic vertex labeling.

Next, we restate the following lemma that appeared in [8].

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Lemma 3 [8] *Let $\mathfrak{A}$ be a sequence $\mathfrak{A} = \{c, c+1, c+2, \dots, c+k\}$, k even. Then there exists a permutation $\Pi(\mathfrak{A})$ of the elements of $\mathfrak{A}$ such that $\mathfrak{A} + \Pi(\mathfrak{A}) = \{2c + \frac{k}{2}, 2c + \frac{k}{2} + 1, 2c + \frac{k}{2} + 2, \dots, 2c + \frac{3k}{2} - 1, 2c + \frac{3k}{2}\}$.*

3 Research Methods and Techniques

Research methods a super (a, d) -edge-antimagic total labeling on disjoint union of cycle non isomorphic are deductive axiomatic and the pattern recognition. Then for research techniques are as follows:

1. calculate the order p and size q on disjoint union of cycle non isomorphic;
2. determine the upper bound for values of d on disjoint union of cycle non isomorphic according by Lemma 1;
3. find label *EAVL* or edge antimagic vertex labeling on disjoint union of cycle non isomorphic;
4. if the label of *EAVL* is *expandable*, then we continue to determine the bijective function of *EAVL*;
5. find label *SEATL* on disjoint union of cycle non isomorphic with feasible values of d ;
6. determine the bijective function of super-edge antimagic total labeling on disjoint union of cycle non isomorphic.

4 Disjoint Union of Cycle Non Isomorphic

Disjoint union of Cycle Non Isomorphic denoted by $C_n \cup C_{n+2} \cup C_{n+4}$, $n \geq 3$ and n odd, is a disconnected graph with vertex set $V = \{x_i^1 \mid 1 \leq i \leq n\} \cup \{x_j^2 \mid 1 \leq j \leq n+2\} \cup \{x_k^3 \mid 1 \leq k \leq n+4\}$, and edge set:

$$E = \{x_i^1 x_{i+1}^1 \cup x_n^1 x_1^1 \mid 1 \leq i \leq n-1\} \cup \{x_j^2 x_{j+1}^2 \cup x_{n+2}^2 x_1^2 \mid 1 \leq j \leq n+1\} \cup \{x_k^3 x_{k+1}^3 \cup x_{n+4}^3 x_1^3 \mid 1 \leq k \leq n+3\}.$$

Thus, $p = q = n + (n+2) + (n+4) = 3n+6$.

If the disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$, has a super (a, d) -edge antimagic total labeling then, for $p = q = 3n+6$, it follows from Lemma 1 that the upper bound of d is $d \leq 3 - \frac{2}{3n+5}$, $d \geq 0$, d is integer, so $d \in \{0, 1, 2\}$

The following theorem describes an $(a, 1)$ -edge antimagic vertex labeling on disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$, $n \geq 3$ and n odd.

Lemma 4 *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has an $(\frac{3n+9}{2}, 1)$ -edge antimagic vertex labeling for $n \geq 3$ and n odd.*

Proof. Define the vertex labeling $\alpha_1 : V(C_n \cup C_{n+2} \cup C_{n+4}) \rightarrow \{1, 2, \dots, 3n+6\}$ in the following way:

For x_i^1 , $1 \leq i \leq n$

$$\alpha_1(x_i^1) = \frac{3i+3}{2} + \left(\frac{(-1)^i+1}{4} \right) 3n$$

For x_j^2 , $1 \leq j \leq n+2$

$$\alpha_1(x_j^2) = \begin{cases} \frac{3j+1}{2} + \left(\frac{(-1)^j+1}{4} \right) 3n + \left(\frac{(-1)^j+1}{2} \right) 3, & \text{if } 1 \leq j \leq n \\ \frac{1}{2} + \left(\frac{(-1)^j+1}{2} \right) 3n + \left(\frac{(-1)^{j+1}+1}{4} \right) 3j + \left(\frac{(-1)^{j+1}}{4} \right) 11, & \text{if } n+1 \leq j \leq n+2 \end{cases}$$

For x_k^3 , $1 \leq k \leq n+4$

$$\alpha_1(x_k^3) = \begin{cases} \frac{3k-1}{2} + \left(\frac{(-1)^k+1}{4} \right) 3n + \left(\frac{(-1)^k+1}{2} \right) 6, & \text{if } 1 \leq k \leq n \\ \frac{3k-1}{2} + \left(\frac{(-1)^k+1}{2} \right) 4n - \left(\frac{(-1)^k+1}{4} \right) 5k + \left(\frac{(-1)^k+1}{4} \right) 13, & \text{if } n+1 \leq k \leq n+4 \end{cases}$$

The vertex labeling α_1 is a bijective function. The edge weights of $C_n \cup C_{n+2} \cup C_{n+4}$ under the labeling α_1 , constitute the following sets:

For i , $1 \leq i \leq n$

$$\begin{aligned} w_{\alpha_1}^1(x_i^1 x_{i+1}^1) &= \frac{3n+6i+9}{2}, & \text{if } 1 \leq i \leq n-1 \\ w_{\alpha_1}^2(x_n^1 x_1^1) &= \frac{3n+9}{2} \end{aligned}$$

For j , $1 \leq j \leq n+2$

$$\begin{aligned} w_{\alpha_1}^3(x_j^2 x_{j+1}^2) &= \frac{3n+6j+11}{2}, & \text{if } 1 \leq j \leq n-1 \\ w_{\alpha_1}^4(x_j^2 x_{j+1}^2) &= \frac{3n+6j+13}{2}, & \text{if } n \leq j \leq n+1 \\ w_{\alpha_1}^5(x_{n+2}^2 x_1^2) &= \frac{3n+11}{2} \end{aligned}$$

For k , $1 \leq k \leq n+4$

$$\begin{aligned} w_{\alpha_1}^6(x_k^3 x_{k+1}^3) &= \frac{3n+6k+13}{2}, & \text{if } 1 \leq k \leq n-1 \\ w_{\alpha_1}^7(x_k^3 x_{k+1}^3) &= \frac{3n+6k+9}{2}, & \text{if } n \leq k \leq n+1 \\ w_{\alpha_1}^8(x_k^3 x_{k+1}^3) &= \frac{3n+6k-1}{2}, & \text{if } n+2 \leq k \leq n+3 \\ w_{\alpha_1}^9(x_{n+4}^3 x_1^3) &= \frac{3n+13}{2} \end{aligned}$$

It is not difficult to see that the set of

$$\bigcup_{t=1}^9 W_{\alpha_1}^t = \left\{ \frac{3n+9}{2}, \frac{3n+11}{2}, \frac{3n+13}{2}, \dots, \frac{9n+19}{2} \right\},$$

consists of consecutive integers. Thus α_1 is a $(\frac{3n+9}{2}, 1)$ -edge antimagic vertex labeling. $\square$

Bača, Y. Lin, M. Miller and R. Simanjuntak (See [9], Theorem 5) have proved that if (p, q) -graph G has an (a, d) -edge antimagic vertex labeling then G has a super $(a+p+q, d-1)$ -edge antimagic total labeling and a super $(a+p+1, d+1)$ -edge antimagic total labeling. With the theorem 1 in hand, and using Theorem 5 from [9], we obtain the following result.

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Proof
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Theorem 1 The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $(\frac{15n+33}{2}, 0)$ -edge antimagic total labeling and a super $(\frac{9n+23}{2}, 2)$ -edge antimagic total labeling, for $n \geq 3$ and n odd.

Proof. Case 1. for $d = 0$

We have proved that the vertex labeling α_1 is a $(\frac{3n+9}{2}, 1)$ -edge antimagic vertex labeling. With respect to Lemma 2, by completing the edge labels $p+1, p+2, \dots, p+q$, we are able to extend labeling α_1 to a super $(a, 0)$ -edge antimagic total labeling, where, for $p = q = 3n+6$, the value $a = \frac{15n+33}{2}$ and common difference 0. Thus a super $(\frac{15n+33}{2}, 0)$ -edge-antimagic total labeling. This concludes the proof. $\square$

Proof. Case 2. for $d = 2$

Label the vertices of $C_n \cup C_{n+2} \cup C_{n+4}$ with $\alpha_2(x_i^1) = \alpha_1(x_i^1)$, $\alpha_2(x_j^2) = \alpha_1(x_j^2)$ and $\alpha_2(x_k^3) = \alpha_1(x_k^3)$, for $i = 1, 2, \dots, n$, $j = 1, 2, \dots, n+2$ and $k = 1, 2, \dots, n+4$; and label the edges with the following way.

For i , $1 \leq i \leq n$

$$\begin{aligned}\alpha_2(x_i^1 x_{i+1}^1) &= 3n + 3i + 7, & \text{if } 1 \leq i \leq n-1 \\ \alpha_2(x_n^1 x_1^1) &= 3n + 7\end{aligned}$$

For j , $1 \leq j \leq n+2$

$$\begin{aligned}\alpha_2(x_j^2 x_{j+1}^2) &= 3n + 3j + 8, & \text{if } 1 \leq j \leq n-1 \\ \alpha_2(x_j^2 x_{j+1}^2) &= 3n + 3j + 9, & \text{if } n \leq j \leq n+1 \\ \alpha_2(x_{n+2}^2 x_1^2) &= 3n + 8\end{aligned}$$

For k , $1 \leq k \leq n+4$

$$\begin{aligned}\alpha_2(x_k^3 x_{k+1}^3) &= 3n + 3k + 9, & \text{if } 1 \leq k \leq n-1 \\ \alpha_2(x_k^3 x_{k+1}^3) &= 3n + 3k + 7, & \text{if } n \leq k \leq n+1 \\ \alpha_2(x_k^3 x_{k+1}^3) &= 3n + 3k + 2, & \text{if } n+2 \leq k \leq n+3 \\ \alpha_2(x_{n+4}^3 x_1^3) &= 3n + 9\end{aligned}$$

The total labeling α_2 is a bijective function from $V(C_n \cup C_{n+2} \cup C_{n+4}) \cup E(C_n \cup C_{n+2} \cup C_{n+4})$ onto the set $\{1, 2, 3, \dots, 6n+12\}$. The edge-weights of $C_n \cup C_{n+2} \cup C_{n+4}$, under the labeling α_2 , constitute the sets:

For i , $1 \leq i \leq n$

$$\begin{aligned}W_{\alpha_2}^1 &= \{w_{\alpha_1}^1 + \alpha_2(x_i^1 x_{i+1}^1), \text{ if } 1 \leq i \leq n-1\} \\ &= (\frac{3n+6i+9}{2}) + (3n + 3i + 7) \\ &= \frac{9n+12i+23}{2}\end{aligned}$$

$$\begin{aligned}W_{\alpha_2}^2 &= \{w_{\alpha_1}^2 + \alpha_2(x_n^1 x_1^1)\} \\ &= (\frac{3n+9}{2}) + (3n + 7) \\ &= \frac{19n+23}{2}\end{aligned}$$

For j , $1 \leq j \leq n+2$

$$\begin{aligned} W_{\alpha_2}^3 &= \{w_{\alpha_1}^3 + \alpha_2(x_j^2 x_{j+1}^2); \text{ if } 1 \leq j \leq n-1\} \\ &= \left(\frac{3n+6j+11}{2}\right) + (3n+3j+8) \\ &= \frac{9n+12j+27}{2} \end{aligned}$$

$$\begin{aligned} W_{\alpha_2}^4 &= \{w_{\alpha_1}^4 + \alpha_2(x_j^2 x_{j+1}^2); \text{ if } n \leq j \leq n+1\} \\ &= \left(\frac{3n+6j+13}{2}\right) + (3n+3j+9) \\ &= \frac{9n+12j+31}{2} \end{aligned}$$

$$\begin{aligned} W_{\alpha_2}^5 &= \{w_{\alpha_1}^5 + \alpha_2(x_{n+2}^2 x_1^2)\} \\ &= \left(\frac{3n+11}{2}\right) + (3n+8) \\ &= \frac{9n+27}{2} \end{aligned}$$

For k , $1 \leq k \leq n+4$

$$\begin{aligned} W_{\alpha_2}^6 &= \{w_{\alpha_1}^6 + \alpha_2(x_k^3 x_{k+1}^3); \text{ if } 1 \leq k \leq n-1\} \\ &= \left(\frac{3n+6k+13}{2}\right) + (3n+3k+9) \\ &= \frac{9n+12k+31}{2} \end{aligned}$$

$$\begin{aligned} W_{\alpha_2}^7 &= \{w_{\alpha_1}^7 + \alpha_2(x_k^3 x_{k+1}^3); \text{ if } n \leq k \leq n+1\} \\ &= \left(\frac{3n+6k+9}{2}\right) + (3n+3k+7) \\ &= \frac{9n+12k+23}{2} \end{aligned}$$

$$\begin{aligned} W_{\alpha_2}^8 &= \{w_{\alpha_1}^8 + \alpha_2(x_k^3 x_{k+1}^3); \text{ jika } n+2 \leq k \leq n+3\} \\ &= \left(\frac{3n+6k-1}{2}\right) + (3n+3k+2) \\ &= \frac{9n+12k+3}{2} \end{aligned}$$

$$\begin{aligned} W_{\alpha_2}^9 &= \{w_{\alpha_1}^9 + \alpha_2(x_{n+4}^3 x_1^3)\} \\ &= \left(\frac{3n+13}{2}\right) + (3n+9) \\ &= \frac{9n+31}{2} \end{aligned}$$

It is not difficult to see that the set of

$$\bigcup_{t=1}^9 W_{\alpha_2}^t = \left\{ \frac{9n+23}{2}, \frac{9n+27}{2}, \frac{9n+31}{2}, \dots, \frac{21n+43}{2} \right\},$$

contains an arithmetic sequence with the first term $\frac{9n+23}{2}$ and common difference 2. Thus α_2 is a super $(\frac{9n+23}{2}, 2)$ -edge antimagic total labeling. This concludes the proof. $\square$

Theorem 2 *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $(6n+14, 1)$ -edge antimagic total labeling for $n \geq 3$ and n odd.*

Proof. We will prove using Lemma 3. For $n \geq 3$ and n odd, consider the vertex labeling α_1 of the graph $C_n \cup C_{n+2} \cup C_{n+4}$ from Lemma 4 which is a $(\frac{3n+9}{2}, 1)$ -EAV

labeling. Let a sequence $\mathfrak{A} = \{c, c+1, c+2, \dots, c+k\}$ be the set of edge-weights of the vertex labeling α_1 for $c = \frac{3n+9}{2}$ and $k = 3n+5$. In Lemma 3, there exists a permutation $\Pi(\mathfrak{A})$ of the elements of $\mathfrak{A}$ such that $\mathfrak{A} + [\Pi(\mathfrak{A}) + \frac{k}{2}] = \{2c+k, 2c+k+1, \dots, 2c+2k\}$. If $[\Pi(\mathfrak{A}) + \frac{k}{2}]$ is an edge labeling of graph $C_n \cup C_{n+2} \cup C_{n+4}$, then $\mathfrak{A} + [\Pi(\mathfrak{A}) + \frac{k}{2}]$ gives the set of the edge weights of $C_n \cup C_{n+2} \cup C_{n+4}$, which implies that the resulting total labeling is super $(6n+14, 1)$. This concludes the proof. $\square$

5 Conclusion

We have lemma and theorem from bijective function of super (a, d) -edge antimagic total labeling on disjoint union of cycle non isomorphic:

- **Lemma 4** *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has an $(a, 1)$ -edge antimagic vertex labeling for $n \geq 3$ and n odd.*
- **Theorem 1** *If $n \geq 3$ and n odd, the disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $(\frac{15n+33}{2}, 0)$ -edge antimagic total labeling and a super $(\frac{9n+23}{2}, 2)$ -edge-antimagic total labeling.*
- **Theorem 2** *The disjoint union of $C_n \cup C_{n+2} \cup C_{n+4}$ has a super $(6n+14, 1)$ -edge antimagic total labeling for $n \geq 3$ and n odd.*

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